The Real Number System

Name Block Date

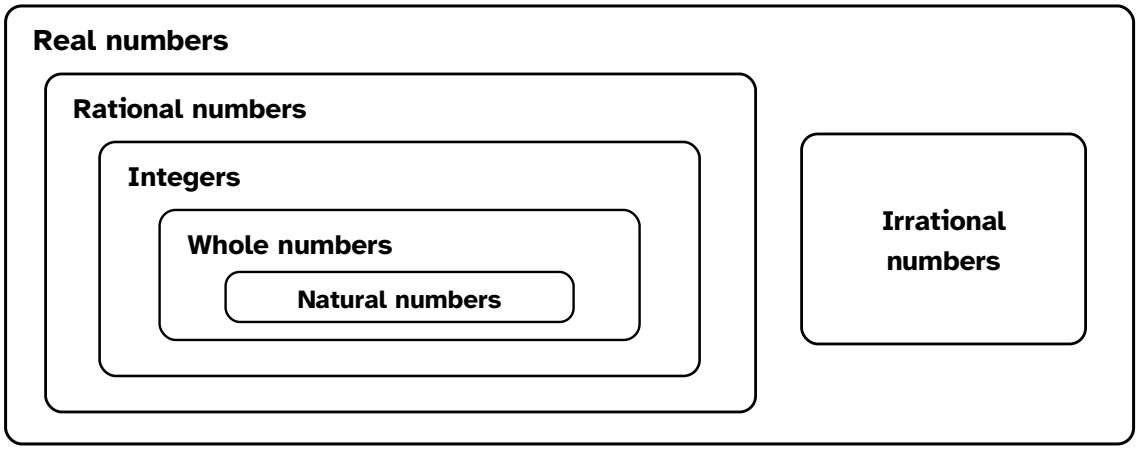
QUESTION 1

Put these numbers into two groups in any way that makes sense to you. Give each group a name. A number can belong to both groups.

-6 0 $\frac{5}{8}$ $\sqrt{49}$ $\sqrt{2}$ π

QUESTION 2

The boxes show how the main sets of real numbers fit together. For each number below, write the smallest box it belongs in.



-12 0 9 $-\frac{4}{5}$

$\sqrt{81}$ $\sqrt{7}$ 3.14159 $0.1010010001\ldots$

QUESTION 3

For each decimal, decide whether it must be rational, must be irrational, or cannot be decided from the information given. Say what made you decide.

a) The decimal stops after 17 digits.

b) After a few digits, the block 407 repeats forever.

c) The zeros between the 1s keep increasing: $0.101001000100001\ldots$

d) A calculator screen shows a decimal that stops at its last display position.

QUESTION 4

Put these numbers in order from least to greatest without changing them to decimals. Use nearby perfect squares to back up your order.

$\sqrt{5}$ $\sqrt{8}$ 3 $\sqrt{10}$ $\sqrt{12}$

QUESTION 5

Dev says, "$\sqrt{12}$ and $\sqrt{3}$ are irrational, so their quotient must be irrational." Dev calculates:

> $$\frac{\sqrt{12}}{\sqrt{3}} = \sqrt{\frac{12}{3}} = \sqrt{4} = 2$$

Check Dev's calculation. What does it show about his claim? Then write a claim about irrational numbers that is actually true.

QUESTION 6

Create two irrational numbers whose sum is rational. Then create two irrational numbers whose sum is irrational. Show enough work to check both examples.

Rational Exponents

Name Block Date

QUESTION 1

Find each value without a calculator.

a) $\sqrt{25}$

b) $\sqrt[3]{8}$

c) $\sqrt[4]{16}$

QUESTION 2

Switch between radical notation and rational exponents. Assume the variables are positive.

a) $x^{3/2}$

b) $y^{2/3}$

c) $\sqrt[5]{p^3}$

d) $(\sqrt[4]{q})^7$

e) $\dfrac{1}{\sqrt[3]{a^2}}$

QUESTION 3

Evaluate without a calculator. Write enough work to show what the numerator, denominator, and negative sign are doing.

a) $27^{2/3}$

b) $16^{3/4}$

c) $32^{2/5}$

d) $81^{-3/4}$

e) $125^{-2/3}$

f) $\left(\frac{64}{125}\right)^{2/3}$

QUESTION 4

For each expression, choose whether you would take the root first or the power first. Evaluate it, then explain which route kept the numbers easier to handle.

a) $64^{2/3}$

b) $16^{3/2}$

c) $81^{3/4}$

QUESTION 5

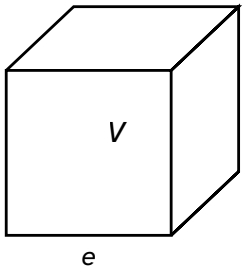
Sienna writes:

> $16^{-3/4} = -\left(\sqrt[4]{16}\right)^3 = -8$

Her root and power are correct, but her final answer is not. Explain what the negative exponent means, correct her work, and check your answer using $16^{3/4}$.

QUESTION 6

A cube has volume *V* and edge length *e*. Write the edge length and total surface area using powers of *V*. Then find both measurements when $V = 216$ cm^3. Finally, explain what happens to the edge length and surface area if the volume is multiplied by 27.



Simplifying Radicals

Name Block Date

QUESTION 1

A square has area 72 cm^2, so its exact side length is $\sqrt{72}$ cm. Noor says the same length can be written as $6\sqrt{2}$ cm.

Square $6\sqrt{2}$ to check Noor's claim. Then use $72 = 36 \cdot 2$ to show how $\sqrt{72}$ becomes $6\sqrt{2}$ without using decimals.

QUESTION 2

Rewrite the number under each radical using a useful square, cube, or fourth power. Then simplify.

a) $\sqrt{48}$

b) $\sqrt{75}$

c) $\sqrt{180}$

d) $\sqrt[3]{54}$

e) $\sqrt[4]{162}$

QUESTION 3

Simplify. Assume all variables are positive. In each answer, make sure nothing else can come out of the radical.

a) $\sqrt{72x^5}$

b) $\sqrt[3]{54a^7}$

c) $\sqrt[4]{80b^9}$

QUESTION 4

Mateo writes:

> $\sqrt{50} = \sqrt{25+25} = \sqrt{25} + \sqrt{25} = 10$

Explain why the middle step can look reasonable even though it is not allowed. Fix the simplification, then square both Mateo's answer and your answer to check which one works.

QUESTION 5

Decide whether each statement is true. Back up each decision without using decimals.

a) $3\sqrt{5} = \sqrt{45}$

b) $2\sqrt{7} = \sqrt{14}$

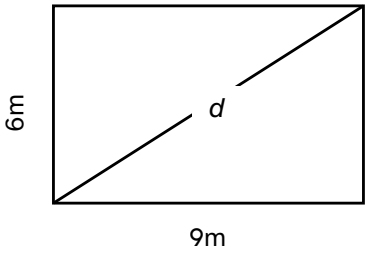
c) $4\sqrt{3} = \sqrt{48}$

d) $2\sqrt[3]{5} = \sqrt[3]{40}$

Use the pattern to write a rule for moving a positive number *k* inside an *n*th root.

QUESTION 6

A rectangular room measures 9 m by 6 m. Find the exact diagonal in simplest radical form. Show that it lies between 10 m and 11 m without using a calculator. A similar room has every length multiplied by a positive scale factor *k*. Find its diagonal and explain what the result says about scaled lengths.



Operations with Radicals

Name Block Date

QUESTION 1

Compare $2\sqrt{3}+5\sqrt{3}$ with $2\sqrt{3}+5\sqrt{2}$. Explain why the first one combines to $7\sqrt{3}$ but the second one does not. What needs to match before radical terms can be combined?

QUESTION 2

Simplify first when needed. Then combine like radical terms.

a) $\sqrt{12}+\sqrt{27}$

b) $3\sqrt{20}-\sqrt{45}$

c) $2\sqrt{18}+\sqrt{8}-\sqrt{50}$

d) $2\sqrt[3]{16}-\sqrt[3]{54}$

QUESTION 3

Before calculating, predict whether each answer will be rational or irrational. Then multiply and simplify.

a) $\sqrt{6} \cdot \sqrt{15}$

b) $(2\sqrt{3})(5\sqrt{6})$

c) $(\sqrt{5}+2)(\sqrt{5}-2)$

d) $(3+\sqrt{2})^2$

Why did the radical terms disappear in part c but not in part d?

QUESTION 4

Harper writes:

> $\sqrt{8} + \sqrt{18} = \sqrt{8 + 18} = \sqrt{26}$

Explain why Harper may have expected radicals to combine this way. Use a quick estimate to show the answer cannot be right, then fix the addition. Can $\sqrt{a} + \sqrt{b} = \sqrt{a + b}$ ever be true when both *a* and *b* are positive?

QUESTION 5

Divide and simplify. Leave no radical in the denominator.

a) $\dfrac{\sqrt{72}}{\sqrt{2}}$

b) $\dfrac{6\sqrt{15}}{3\sqrt{5}}$

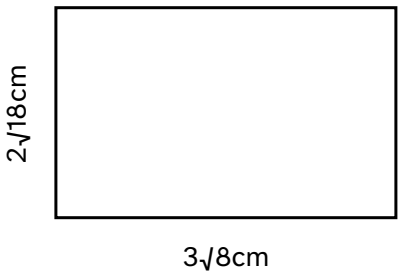
c) $\dfrac{5}{\sqrt{2}}$

d) $\dfrac{3}{2\sqrt{3}}$

Why does multiplying by $\dfrac{\sqrt{a}}{\sqrt{a}}$ change the form but not the value? What must be true about *a*?

QUESTION 6

A rectangle has the exact dimensions shown. Decide whether it is actually a square. Then find its exact perimeter and area. Explain why one answer is irrational while the other is rational.

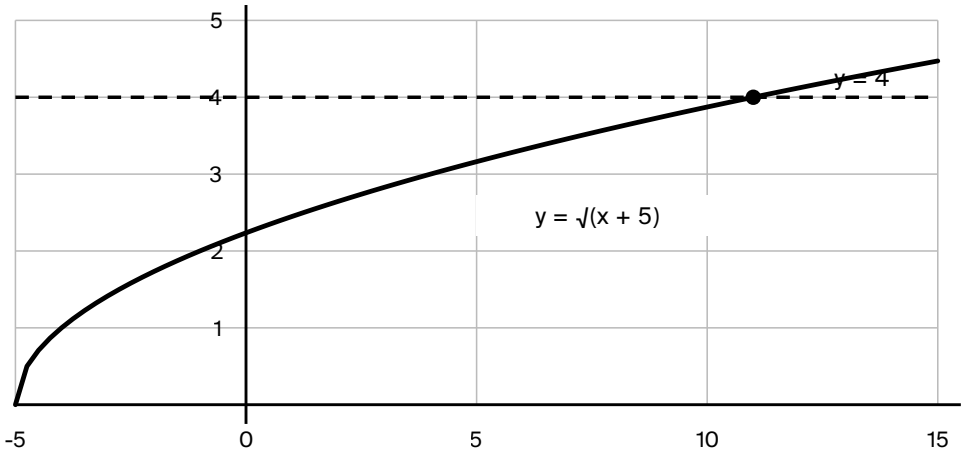


Radical Equations

Name Block Date

QUESTION 1

The graph shows $y = \sqrt{x+5}$ and $y = 4$. Explain why the *x*-coordinate of their intersection solves $\sqrt{x+5} = 4$. Then solve the equation algebraically and connect your steps to the graph.



QUESTION 2

Solve each equation. Isolate the radical before squaring, and check your result in the original equation.

a) $\sqrt{2x+3} = 7$

b) $\sqrt{x-4} + 2 = 8$

c) $3\sqrt{x+1} - 2 = 10$

QUESTION 3

Solve $\sqrt{x+6} = x$. Before squaring, record what must be true about *x*. After solving the resulting quadratic equation, check every candidate in the original equation and explain why any rejected value appeared.

QUESTION 4

Alina writes:

> $\sqrt{x+1} = x-1 \Rightarrow x+1 = (x-1)^2 \Rightarrow x^2 - 3x = 0 \Rightarrow x = 0, 3$
>
> She reports both values as solutions.

Check Alina's algebra and both candidates. What was missing from her solution, and why can squaring create a candidate that does not work?

QUESTION 5

Decide whether each claim is always, sometimes, or never true. Give a reason or a counterexample.

a) A real solution makes every square-root radicand non-negative.

b) Every solution found after squaring solves the original equation.

c) An equation of the form $\sqrt{ax+b}=c$, with $c<0$, has a real solution.

d) Checking candidates may remove candidates, but it cannot create new ones.

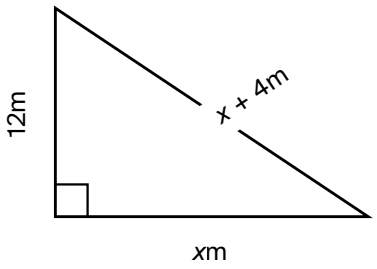
QUESTION 6

A right triangular support has the dimensions shown.

a) Write a radical equation for x and predict whether x is shorter or longer than 12 m.

b) Solve the equation and find all three side lengths.

c) Check the measurements using the Pythagorean theorem.

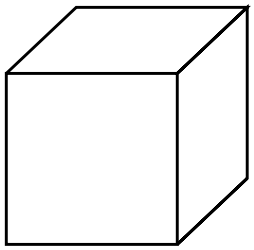


Powers and Radicals Word Problems

Name Block Date

QUESTION 1

A cube-shaped storage bin holds 512 L. One litre is 1 dm^3. The company needs the edge length and the total outside area before ordering material.



e

Organize the problem before solving it.

What we know

What we need

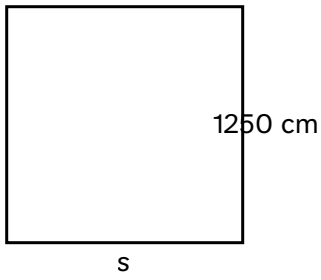
Useful relationships

Units to report

Now find both measurements and check that they make sense for the stated volume.

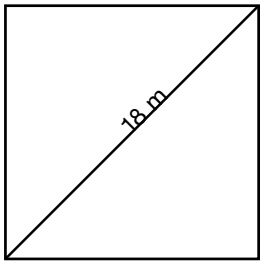
QUESTION 2

A square mosaic has area 1250 cm^2. A metal strip will run around its entire edge. Find the exact side length and exact strip length. Then give the strip length to the nearest tenth of a centimetre.



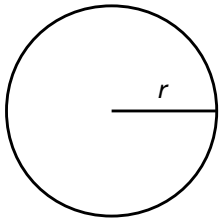
QUESTION 3

A square courtyard has a diagonal of 18 m. Find the exact side length and the perimeter. Then give the perimeter to the nearest tenth of a metre.



QUESTION 4

A circular community garden has area 450π m^2. Fencing will go around the whole garden. Find the exact radius and exact fence length. Then give the fence length to the nearest tenth of a metre.



QUESTION 5

Two cube-shaped shipping boxes are similar. The larger box has 64 times the volume of the smaller box. The smaller box has an edge length of 12 cm. Find the larger edge length. Then compare the amount of cardboard needed for the two closed boxes by finding the ratio of their surface areas. Explain why the surface-area ratio is not 64.

Common Factors and Structure

Name Block Date

QUESTION 1

Expand $6x(2x+3)$. Then factor $12x^2+18x$. Explain how your two calculations undo each other.

QUESTION 2

Factor each polynomial by taking out its greatest common factor. Expand your answer for one of them as a check.

a) $8x+12$

b) $15a^2-20a$

c) $18m^3n+24m^2n^2$

d) $14p^3-21p^2+7p$

QUESTION 3

Factor completely. If the first term is negative, decide whether taking out a negative factor makes the result easier to read.

a) $-14x^2 + 21x$

b) $9r^4s^2 - 12r^2s^3$

c) $-10y^3 - 25y^2 + 15y$

d) $16u^2v - 24uv^2 + 8uv$

QUESTION 4

Leah and Owen factor the same polynomial.

Leah

$12x^2 + 18x = 3x(4x + 6)$

Owen

$12x^2 + 18x = 6x(2x + 3)$

Expand both answers. Explain why both are equivalent to the original polynomial, but only one has removed the greatest common factor. How can you tell whether a common factor remains?

QUESTION 5

Decide whether each polynomial has been factored completely using common factors. If it has not, finish the job.

a) $6x^2 + 9x = 3x(2x + 3)$

b) $20a^2 - 30a = 5a(4a - 6)$

c) $-12m^3 - 18m^2 = -6m^2(2m + 3)$

d) $14p^3 - 21p^2 + 35p = 7p(2p^2 - 3p + 5)$

QUESTION 6

A concert has $15x^2 + 25x$ seats arranged in $5x$ equal rows. Factor the total to find the number of seats in each row. When $x = 4$, find the number of rows, the seats in each row, and the total number of seats. Multiply to check.

Factoring Polynomials Starting with x Squared

Name Block Date

QUESTION 1

Expand each product. Keep the two middle products separate until the last line.

a) $(x+2)(x+5)$

b) $(x-3)(x-4)$

c) $(x+6)(x-2)$

For each result, compare the two numbers in the factors with the middle coefficient and the constant term. What two calculations keep appearing?

QUESTION 2

Factor each polynomial. Expand every answer to check both the middle term and the constant term.

a) $x^2 + 9x + 20$

b) $x^2 + 11x + 24$

c) $x^2 - 9x + 20$

d) $x^2 + x - 20$

QUESTION 3

Factor completely.

a) $x^2 + 7x + 10$

b) $x^2 + 13x + 36$

c) $x^2 - 2x - 35$

d) $x^2 - 12x + 36$

e) $x^2 - 5x - 24$

f) $x^2 + 2x - 48$

QUESTION 4

Predict the signs in the factors before you work out the numbers. Then factor.

a) $x^2 + 8x + 15$

b) $x^2 - 8x + 15$

c) $x^2 + 2x - 15$

d) $x^2 - 2x - 15$

QUESTION 5

Mira writes:

> $x^2 + 7x + 12 = (x + 2)(x + 6)$
>
> “Two times six is twelve, so the factors work.”

Expand Mira's factors and check her work. Explain what she forgot to check, then write and verify the correct factorization.

QUESTION 6

A rectangular mural has area $x^2 + 5x - 24$ m^2. One side is $x + 8$ m. Find the other side and the values of *x* that make both side lengths positive. Then write the perimeter in simplest form and use expansion to check the area.

Factoring Polynomials

Name Block Date

QUESTION 1

Jules thinks that since 3 and 4 multiply to 12, this should work:

> $2x^2 + 11x + 12 = (x + 3)(x + 4)$

a) Expand out Jules' factors and check his work. What doesn't match the original?

b) What did Jules forget to account for?

QUESTION 2

Factor $2x^2 + 11x + 12$. Show how you split the middle term, then group. Multiply your factors to check.

QUESTION 3

Factor each polynomial. Multiply your factors to check.

a) $3x^2 + 10x + 8$

b) $2x^2 - x - 6$

c) $6x^2 + 11x + 3$

QUESTION 3 CONTINUED

d) $4x^2 - 4x - 15$

e) $5x^2 - 13x + 6$

f) $8x^2 + 2x - 3$

QUESTION 4

Sam factors $6x^2 + 7x + 2$:

> $6x^2 + 7x + 2 = (6x + 1)(x + 2)$
>
> Sam says, “One times two gives the constant term, so these factors work.”

a) What part of Sam's answer matches the original?

b) Multiply Sam's factors. What doesn't match?

c) Sam only looked at the 2. What did that make him miss?

d) Fix Sam's answer. Multiply your factors to check.

QUESTION 5

Factor $18p^3 + 15p^2 - 12p$ as far as you can. How can you tell you're done?

QUESTION 6

A fundraiser earns $2x^2 + 7x + 6$ dollars. The money is split equally among $x + 2$ teams. Factor the total to find each team's share. Then let $x = 1$ and find the number of teams, each team's share, and the total. Multiply to check.

Difference of Squares and Choosing a Method

Name Block Date

QUESTION 1

Noor wants to calculate $47 \cdot 53$ mentally. She rewrites it as $(50-3)(50+3)$. Predict whether the result is greater or less than 50^2. Then calculate the product without long multiplication and explain which terms cancel.

QUESTION 2

Factor completely.

a) $x^2 - 25$

b) $9x^2 - 16$

c) $49a^2 - 81b^2$

d) $3x^2 - 75$

e) $8a^3 - 18a$

f) $x^4 - 16$

QUESTION 3

Factor the three expressions. Then explain how you can quickly tell a repeated factor from a pair of conjugate factors.

a) $x^2 + 10x + 25$

b) $x^2 - 25$

c) $9x^2 - 24x + 16$

QUESTION 4

Priya writes:

> $x^2 + 16 = (x + 4)(x - 4)$
>
> “Both terms are squares, so the pattern works.”

Expand Priya's factors and check her claim. Explain why the sign between the squares matters, and state what can be said about factoring $x^2 + 16$ into real linear factors.

QUESTION 5

Factor completely. For each one, decide whether the first useful move is a common factor, a factor pair, a difference of squares, or a perfect-square pattern.

a) $12x^3 - 27x$

b) $6x^2 + 7x + 2$

c) $x^2 - 14x + 49$

d) $5x^2 - 45$

e) $2x^2 + 9x + 10$

f) $4x^2 + 12x + 9$

QUESTION 6

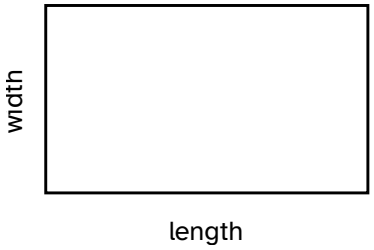
Create a polynomial that needs a common factor first and a difference of squares second. Factor it completely, then expand your answer to check it.

Factoring Word Problems

Name Block Date

QUESTION 1

A rectangular patio has area $6x^2 + 17x + 5$ m^2. Its dimensions are polynomial expressions.



Organize the problem before solving it.

What we know

What we need

Useful relationship

How to check

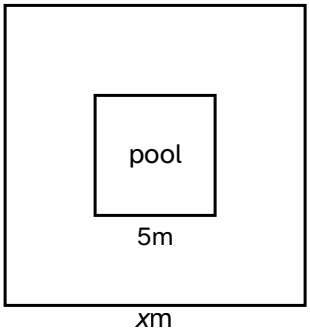
Factor the area to find possible dimensions. When $x = 3$, find the dimensions, area, and perimeter.

QUESTION 2

A school orders $2x^2 + 11x + 5$ badges and shares them equally among $2x + 1$ classes. Factor the total to find how many badges each class receives. When $x = 4$, find the number of classes, the badges per class, and the total. Multiply to check.

QUESTION 3

A square courtyard has side length *x* metres. A centred square pool has side length 5 m. The remaining courtyard area is 144 m^2. Find the courtyard side length and the uniform width of the space around the pool.



QUESTION 4

Two consecutive positive locker numbers have a product of 156. Find the two locker numbers. Show how factoring lets you reject the pair that does not fit the situation.

QUESTION 5

A rectangular stage has area 120 m^2. Its length is 2 m greater than its width. Find both dimensions and check the area. Explain why the negative solution from the factored equation does not make sense here.

Recognizing Quadratic Relationships

Name Block Date

QUESTION 1

The three columns show different relationships. Before calculating differences, write down anything you notice. Then classify each relationship as linear, quadratic, or neither.

x	**A**	**B**	**C**
-2	1	4	2
-1	3	1	4
0	5	0	8
1	7	1	16
2	9	4	32

QUESTION 2

Use differences to decide whether this table is quadratic. Predict the value when $x = 5$, and explain how the differences support your answer.

x	**0**	**1**	**2**	**3**	**4**	**5**
***f*(*x*)**	3	6	11	18	27	
first difference						
second difference						

QUESTION 3

Decide which equations define quadratic functions. For each quadratic, state whether it opens up or down.

a) $y = 3x - 2$

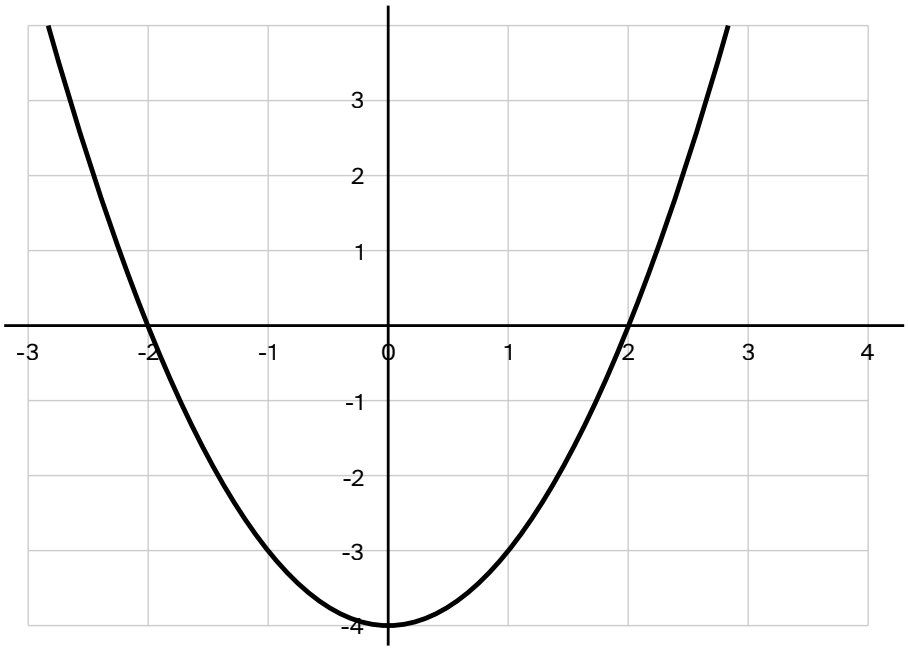
b) $y = -2x^2 + 5x + 1$

c) $y = 2^x$

d) $y = (x - 3)(x + 1)$

QUESTION 4

Read the graph. Find the vertex, axis of symmetry, *x*-intercepts, *y*-intercept, domain, and range.



QUESTION 5

Lia looks at the values $4, 6, 10, 16, 24$ for equally spaced x-values. She says, “The first differences are not equal, so this cannot be quadratic.”

Check her conclusion. What did she overlook, and what should the next value be if the pattern continues?

QUESTION 6

For $x = -2, -1, 0, 1, 2$, a relationship has values $-3, 3, 5, 3, -3$. Find the first and second differences. What does the sign of the second differences tell you? Sketch the graph.

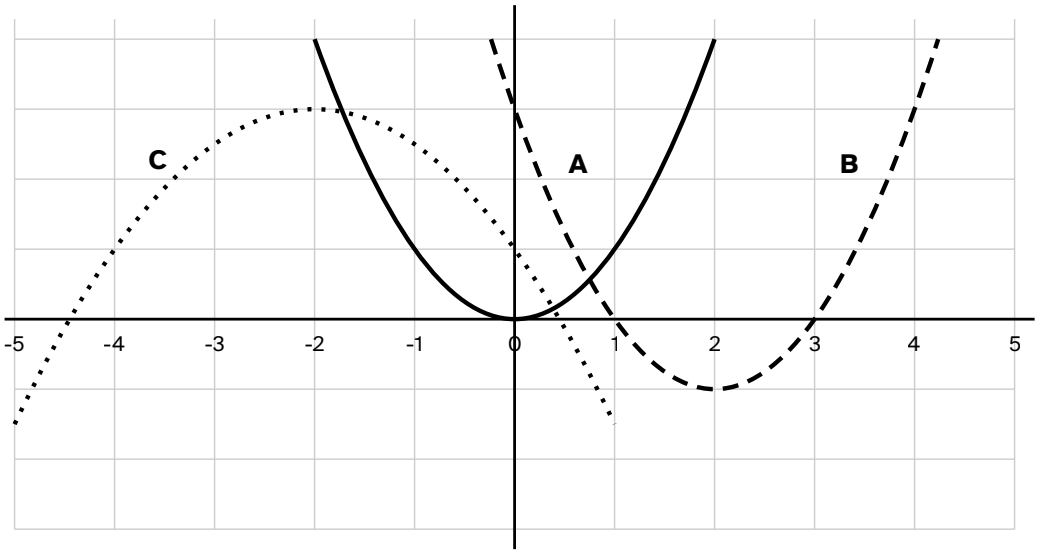
Transformations and Vertex Form

Name Block Date

QUESTION 1

Match curves A, B, and C with the equations below. For each curve, state its vertex and whether it opens up or down.

$y = x^2$ $y = (x-2)^2 - 1$ $y = -\frac{1}{2}(x+2)^2 + 3$



QUESTION 2

Describe how each graph is changed from $y = x^2$.

a) $y = 2(x-3)^2 + 1$

b) $y = -(x+4)^2$

c) $y = \frac{1}{3}(x-2)^2 - 5$

d) $y = -3x^2 + 2$

QUESTION 3

Write each equation in vertex form using the vertex and one other point.

a) vertex $(2, -3)$, point $(4, 5)$

b) vertex $(-1, 4)$, point $(1, -4)$

c) vertex $(0, 1)$, point $(3, 10)$

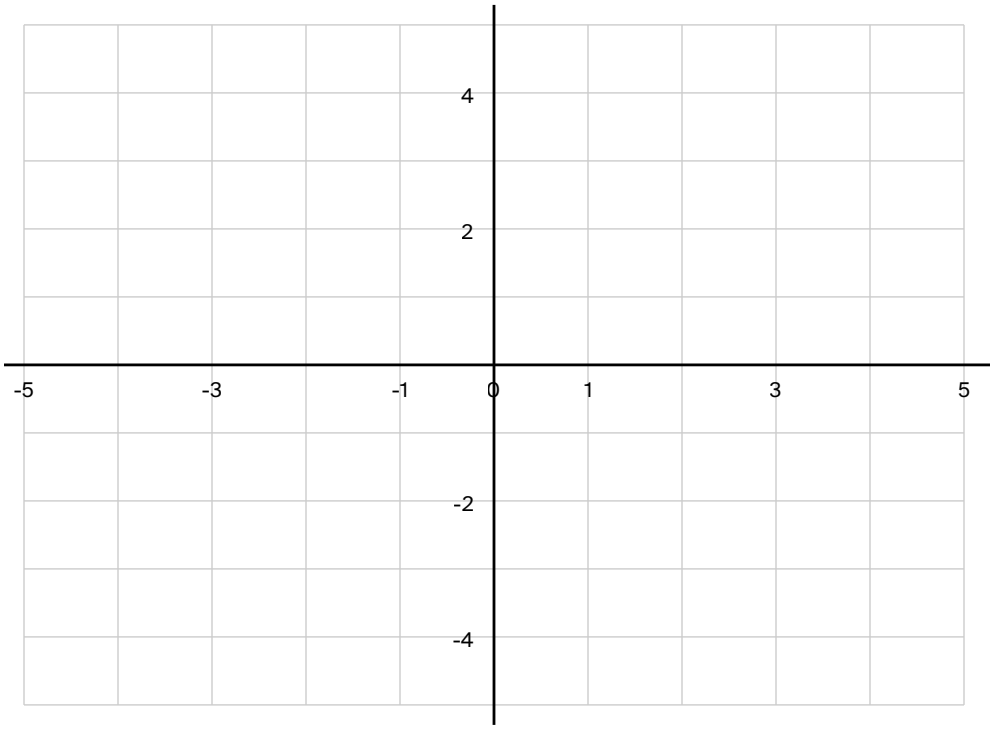
QUESTION 4

Harper says $y = (x + 3)^2 - 2$ is $y = x^2$ moved right 3 and down 2. Check the claim by finding the vertex and substituting one easy *x*-value. Explain the horizontal shift clearly.

QUESTION 5

Sketch both functions on the same coordinate grid. Label each vertex and one symmetric pair of points.

a) $y = (x-1)^2 - 4$ **b)** $y = -2(x+2)^2 + 3$



QUESTION 6

Create a quadratic function with vertex $(3, -5)$ that opens down and is narrower than $y = x^2$.
Give one point on your graph and explain how you know it works.

Connecting Quadratic Forms

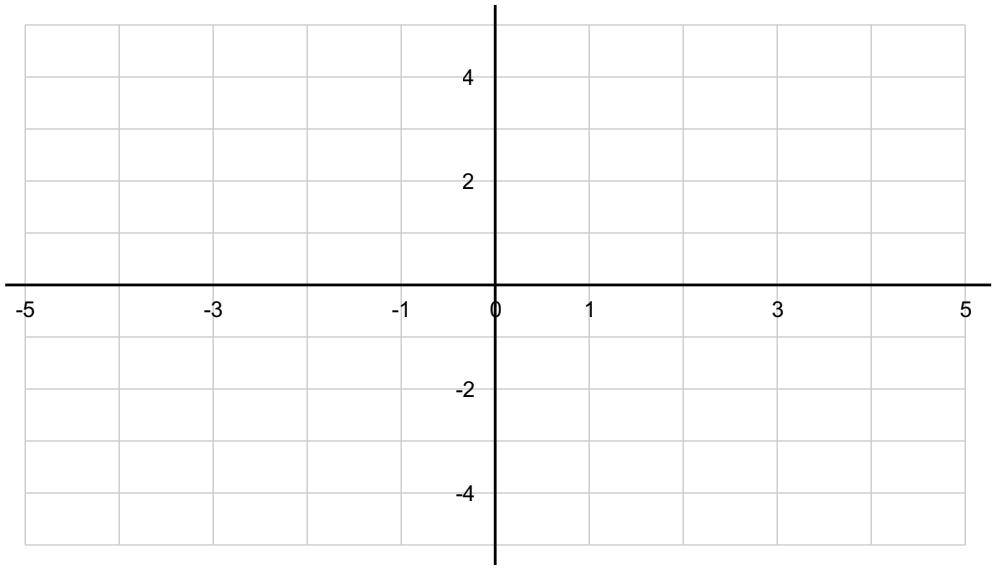
Name Block Date

QUESTION 1

These three equations describe the same graph: $f(x) = x^2 - 2x - 3$, $f(x) = (x-3)(x+1)$, and $f(x) = (x-1)^2 - 4$. Circle the form you would use first to find the zeros, the vertex, and the *y*-intercept. Then list those features.

QUESTION 2

Sketch that graph. Label the vertex and all intercepts.



QUESTION 3

Rewrite each equation in the requested form.

a) $(x-2)(x+6)$ to standard form

b) $2(x-1)^2-8$ to standard and factored form

c) x^2+6x+5 to factored and vertex form

QUESTION 4

Noah says $(x-4)(x+2)$ has vertex $(4,-2)$ because those are the numbers he sees. Find the actual vertex. What do the 4 and -2 tell you?

QUESTION 5

Choose the form you would use first to answer each question about $g(x) = 2x^2 - 12x + 10 = 2(x-1)(x-5) = 2(x-3)^2 - 8$. Explain each choice in a short sentence.

Where does the graph cross the *x*-axis?

What is its minimum value?

Where does it cross the *y*-axis?

QUESTION 6

Make one quadratic in standard, factored, and vertex form. Use the forms to give its zeros, vertex, and *y*-intercept. Check that all three equations are equivalent.

Quadratic Function Word Problems

Name Block Date

QUESTION 1

A toy rocket follows $h(t) = -5(t-2)^2 + 20$, where *h* is height in metres and *t* is time in seconds. Find its greatest height, when it reaches that height, and when it is on the ground.

What do you know?

What are you finding?

Which form helps?

What will you check?

QUESTION 2

A community garden has 60 m of fencing for a rectangle. If one side is *x* metres, its area is $A(x) = x(30 - x)$. Find the dimensions that give the greatest area. Explain why your answer makes sense.

QUESTION 3

A bridge arch is modelled by $y = -\frac{1}{4}(x-6)^2 + 9$, with distances in metres. The road is at $y = 0$. Find the height and width of the opening.

QUESTION 4

A small theatre models its ticket revenue by $R(p) = -50(p-18)^2 + 16200$, where *p* is the ticket price in dollars. What ticket price gives the greatest revenue? What is that revenue? Explain how the equation shows both answers.

QUESTION 5

A soccer ball is kicked from a 1 m platform. Its height is $h(t) = -5t^2 + 20t + 1$. Find the greatest height. Then find the two times when the ball is 16 m high and say what those times mean.

Zeros, Factors, and Graphs

Name Block Date

QUESTION 1

Without solving anything new, connect these three statements: $(x-5)(x+2)=0$, the zeros are 5 and -2, and the graph crosses the *x*-axis at $(5,0)$ and $(-2,0)$. What stays the same?

QUESTION 2

Solve each equation by factoring. Check each answer in the original equation.

a) $x^2-3x-10=0$

b) $2x^2+5x-3=0$

c) $3x^2-12x=0$

d) $x^2+4=0$

QUESTION 3

The graph of $y = (x + 4)(x - 2)$ has two *x*-intercepts. Find them, the axis of symmetry, and the vertex.

QUESTION 4

Mika solves $x(x - 7) = 18$ by writing $x = 0$ or $x = 7$. What went wrong? Fix the solution.

QUESTION 5

Find all values of *k* that make $x^2 + kx + 16 = 0$ factor into two equal factors. Explain how the graph would look in each case.

QUESTION 6

Write a quadratic equation with zeros -3 and 7 that also passes through (1,-24). Show how you chose the leading number.

Square Roots and Completing the Square

Name Block Date

QUESTION 1

Compare $(x-3)^2 = 16$ and $x - 3 = 4$. How many answers does each equation have? Explain the difference.

QUESTION 2

Solve by taking square roots.

a) $(x+1)^2 = 25$

b) $(x-3)^2 = 12$

c) $4x^2 = 25$

d) $2(x-4)^2 + 6 = 0$

QUESTION 3

Complete the square, then write each expression in vertex form.

a) $x^2 + 8x + 5$

b) $2x^2 - 12x + 7$

c) $-x^2 - 4x + 9$

QUESTION 4

Solve $x^2 + 6x - 7 = 0$ by completing the square. Then check your answers by factoring.

QUESTION 5

Ari writes $x^2 - 10x + 7 = (x - 5)^2 + 7$. Expand the right side and check it. Then fix the vertex form.

QUESTION 6

Choose a value of c so $x^2 + 12x + c$ is a perfect square. Then choose a different value of c so the expression has no real zeros. Explain both choices.

Quadratic Formula and Choosing a Method

Name Block Date

QUESTION 1

For each equation, choose the method you would try first: factoring, square roots, completing the square, or the quadratic formula. Do not solve yet. Give one reason for each choice.

a) $x^2 - 9x + 20 = 0$

b) $3(x + 2)^2 = 21$

c) $2x^2 + 3x - 4 = 0$

d) $x^2 + 6x + 1 = 0$

QUESTION 2

Use the quadratic formula. Give exact answers, then decimal answers to the nearest hundredth.

a) $2x^2 + 3x - 4 = 0$

b) $x^2 - 4x - 3 = 0$

c) $3x^2 + 2x + 5 = 0$

QUESTION 3

Without solving, use the discriminant to say whether each equation has two, one, or no real solutions.

a) $x^2 + x - 12 = 0$

b) $4x^2 + 12x + 9 = 0$

c) $x^2 - 2x + 5 = 0$

QUESTION 4

Jules uses $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ on $-2x^2 + 5x + 3 = 0$ but sets $a = 2$. Follow that choice far enough to see the problem, then solve correctly.

QUESTION 5

Solve each equation using an efficient method. Name the method you chose.

a) $x^2 - 7x + 12 = 0$

b) $(x - 3)^2 = 5$

c) $3x^2 + x - 5 = 0$

QUESTION 6

Compare $(x - 2)(x + 5) = 0$ and $x^2 + 3x = 10$. Show that they have the same solutions. Which equation is ready to solve first?

Quadratic Equation Word Problems

Name Block Date

QUESTION 1

A rectangular mural has area 96 m². Its length is 4 m more than its width. Find its dimensions.

What do you know?

What are you finding?

Choose a variable

Write a useful equation

QUESTION 2

A ball is thrown upward from a balcony. Its height is $h(t) = -5t^2 + 15t + 20$ metres. When does it hit the ground? Ignore any time before it was thrown.

QUESTION 3

Two consecutive positive odd integers have a product of 143. Find the integers.

QUESTION 4

A photo is 18 cm by 24 cm. A border of equal width is placed inside all four edges, leaving 280 cm^2 of visible photo. Find the border width.

QUESTION 5

A right triangle has legs that differ by 7 cm and a hypotenuse of 13 cm. Find the two leg lengths.

QUESTION 6

A rectangular dog run uses a wall as one side and 34 m of fencing for the other three sides.
What dimensions give an area of 120 m^2? There are two possible runs. Find both.

Restrictions and Equivalent Expressions

Name Block Date

QUESTION 1

Compare $\frac{x^2-9}{x-3}$ and $x+3$. Try $x=4$ and $x=3$ in both. In what way are they the same, and what is different?

QUESTION 2

State every restriction before simplifying.

a) $\frac{x^2-16}{x-4}$

b) $\frac{x(x-2)}{x(x+5)}$

c) $\frac{(x+3)^2}{x^2-9}$

QUESTION 3

Simplify. Keep the restrictions from the original expression.

a) $\frac{x^2-1}{x^2+x-2}$

b) $\frac{2x^2-12x+18}{x^2+x-12}$

c) $\frac{x^2-10x+25}{x^2-25}$

QUESTION 4

Sam crosses out the *x* in $\frac{x+6}{x}$ and writes 6. Use $x = 2$ to check the claim. Then explain when cancelling is allowed.

QUESTION 5

Find values of *a* and *b* so $\frac{x^2+ax+b}{x+2}$ simplifies to $x+5$, with one restriction. Show how you know.

QUESTION 6

Compare $\frac{(x-1)(x+2)}{x-1}$ and $\frac{(x-3)(x+2)}{x-3}$. What do both simplify to? Why are the original expressions still different?

Operations and Equations

Name Block Date

QUESTION 1

Which pair can be added without changing either denominator: $\frac{2}{x+1} + \frac{5}{x+1}$ or $\frac{2}{x+1} + \frac{5}{x-1}$?
Explain what the denominator is doing.

QUESTION 2

Simplify each result and state restrictions.

a) $\frac{3}{x+2} + \frac{4}{x+2}$

b) $\frac{2}{x-1} + \frac{3}{x+1}$

c) $\frac{x^2-1}{x^2-2x} \cdot \frac{x}{x-1}$

d) $\frac{\frac{3x}{2x-2}}{\frac{x}{x+2}}$

QUESTION 3

Solve. Check each answer against the original restrictions.

a) $\frac{3}{x} + \frac{1}{4} = \frac{5}{8}$

b) $\frac{2}{x-1} = \frac{4}{x+1}$

c) $\frac{1}{x-2} + \frac{1}{x+2} = \frac{10}{21}$

QUESTION 4

Priya solves $\frac{x+2}{x-3} = 1$ by cancelling the x terms and getting $\frac{2}{-3} = 1$. Check the original equation with a few values, then solve it correctly.

QUESTION 5

Solve $\frac{x}{x-2} = \frac{6}{x^2-4} + 1$. State restrictions first and check the result.

QUESTION 6

Find a value of *k* so $\frac{3}{x-1} + \frac{k}{x+1} = \frac{5x+1}{x^2-1}$ is true for every allowed *x*. Explain how you found it.

Rational Expression Word Problems

Name Block Date

QUESTION 1

One hose fills a tank in 6 hours. A second hose fills it in 4 hours. How long do they take together?

What is each rate?

What rate is needed?

Write an equation

What will the answer mean?

QUESTION 2

A paddler travels 12 km downstream and 12 km upstream. The river current is 2 km/h. The whole trip takes 5 hours. Find the paddler's speed in still water.

QUESTION 3

A printer can finish a job in 30 minutes. Working with a second printer, the job takes 18 minutes. How long would the second printer take alone?

QUESTION 4

A cyclist rides 24 km to a park, then returns along the same route at a speed 4 km/h faster. The return trip takes 30 minutes less. Find both speeds.

QUESTION 5

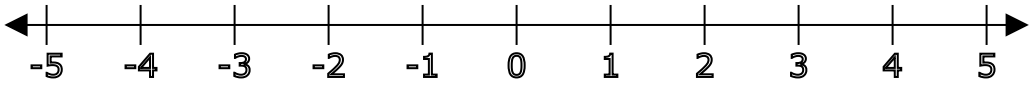
A class buys a $360 piece of equipment. If 6 more students join, each person pays $5 less. How many students were in the original group?

Linear Inequalities and Interval Notation

Name Block Date

QUESTION 1

Which values from $\{-3, -1, 0, 2, 5\}$ make $2x - 1 < 5$ true? Put them on the number line. What boundary separates the values that work from the values that do not?



QUESTION 2

Solve each inequality. Give the answer in inequality notation and interval notation.

a) $3x + 4 < 19$

b) $-2x + 5 \geq -3$

c) $-7 < 2x - 1 \leq 3$

d) $x \leq -2$ or $x > 3$

QUESTION 3

Write each number-line description in interval notation.

a) from -1, included, to 4, not included

b) less than 2 or greater than 6

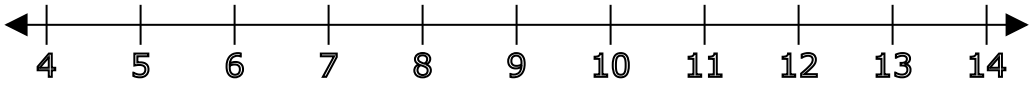
c) at least 3

QUESTION 4

Eli solves $-4x > 12$ and writes $x > -3$. Test one value from Eli's answer, then fix it. Why does the sign change direction?

QUESTION 5

Solve $2 < \frac{x+1}{3} \leq 5$. Show the answer on a number line and in interval notation.



QUESTION 6

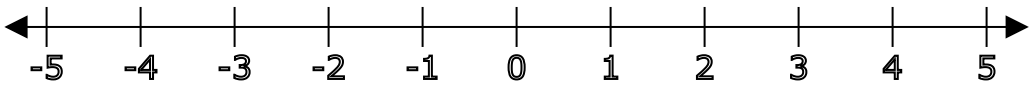
Compare $-2 \leq x < 5$ and $0 \leq x + 2 < 7$. Do they have the same solution? Show how you know and give the interval.

Quadratic Inequalities and Sign Analysis

Name Block Date

QUESTION 1

The factors in $(x-2)(x+3)$ change sign at -3 and 2. Pick one number from each interval and decide whether the product is positive or negative.



QUESTION 2

Solve each inequality. Show the boundary values and give interval notation.

a) $(x-2)(x+3) > 0$

b) $x^2 - 3x - 4 \leq 0$

c) $2x^2 - 2x - 12 \geq 0$

QUESTION 3

Solve. Factor first when it helps.

a) $x^2 - 3x - 10 < 0$

b) $2x^2 - 5x - 3 \geq 0$

c) $x^2 + 4x + 7 > 0$

QUESTION 4

Zoey solves $x^2 < 9$ by taking square roots and writing $x < 3$. Find a value that exposes the problem, then give the full answer.

QUESTION 5

Find all values of *k* for which $x^2 - 6x + k > 0$ is true for every real *x*. Explain using either the vertex or the discriminant.

QUESTION 6

Draw a parabola whose solution to $f(x) \leq 0$ is $[-4, 1]$. Write one possible equation for it.

Inequality Word Problems

Name Block Date

QUESTION 1

A student can spend at most $42 on a ride-share. The trip costs $6 plus $2.40 per kilometre. How far can the student travel?

What is fixed?

What changes?

What does “at most” mean?

Write an inequality

QUESTION 2

A rectangular pen uses 40 m of fencing. One side is x metres, so its area is $A = x(20 - x)$. For which side lengths is the area at least 75 m²?

QUESTION 3

A small business models daily profit by $P(n) = -2(n-10)(n-40)$, where n items are sold. For what sales numbers is the profit positive? Remember that sales must be whole numbers.

QUESTION 4

The stopping distance of a bicycle is modelled by $d = 0.05v^2 + 0.3v$, where v is speed in km/h and d is metres. A path has 14 m of clear space. Find the greatest safe speed to the nearest km/h.

QUESTION 5

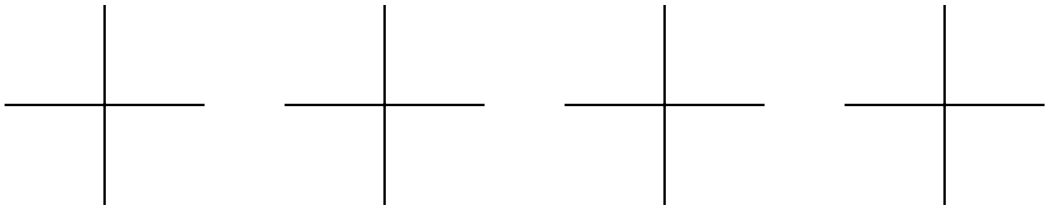
A concert hall earns $R(p) = -100(p - 25)^2 + 62500$ dollars when tickets cost p dollars. The event needs at least \$60,000 in revenue. Find the allowed ticket-price range.

Angles in Standard Position

Name Block Date

QUESTION 1

Start on the positive *x*-axis. For each turn, say which quadrant or axis the terminal arm reaches. Sketch each turn on its axes.



80° 170° 270° -45°

QUESTION 2

Find one positive and one negative coterminal angle for each angle.

a) 30°

b) -225°

c) 600°

QUESTION 3

Find the reference angle and quadrant.

a) 145°

b) 220°

c) -30°

d) 555°

QUESTION 4

Kai says the reference angle for -140° is 140° because reference angles are positive. Sketch the angle and fix the answer.

QUESTION 5

An angle in standard position has a reference angle of 28°. Find every possible angle from 0° to 360°.

QUESTION 6

Compare 40° and 400°. Draw both turns. Are they coterminal? Explain without relying only on your drawing.

Radian Measure

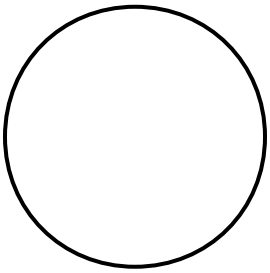
Name Block Date

QUESTION 1

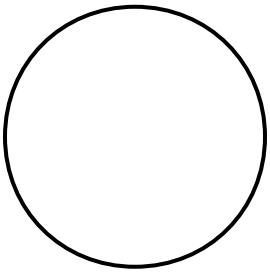
Why do you think people settled on 360 degrees for one full turn? What makes 360 convenient? Write down two useful features and one drawback.

QUESTION 2

The circles show the same full turn in two systems. Degrees use 360. Radians use 2π. What do you notice before doing any conversions?



360°



2π radians

QUESTION 3

A radian compares arc length with radius. Imagine wrapping pieces, each one radius long, around a circle. A circle has circumference $2\pi r$. How many of those pieces fit? Use your answer to explain why a full turn is 2π radians.

QUESTION 4

Build the common conversions by starting with a full turn and repeatedly taking halves.

0° = ____

360° = ____

180° = ____

90° = ____

45° = ____

QUESTION 5

Convert each angle. Keep radian answers exact.

a) 30°

b) 120°

c) 225°

d) $\frac{5\pi}{3}$

e) $\frac{7\pi}{4}$

QUESTION 6

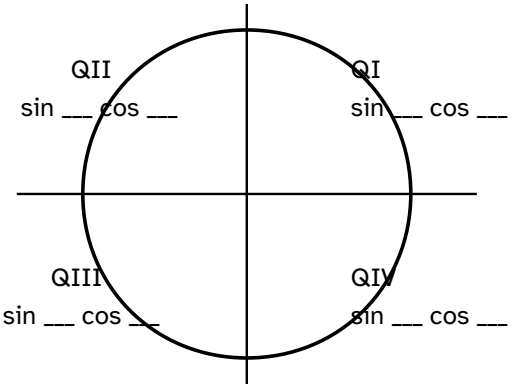
A student says $60^\circ = \frac{\pi}{60}$. Use the full-turn relationship to show why that cannot be right, then fix it.

Special Angles and the Unit Circle

Name Block Date

QUESTION 1

On a unit circle, a point has coordinates $(cos\,\theta, sin\,\theta)$. Use the signs of *x* and *y* to fill in the sign of sine and cosine in each quadrant.



QUESTION 2

Find the exact sine, cosine, and tangent.

a) $30°$

b) $135°$

c) $\frac{4\pi}{3}$

d) $\frac{3\pi}{2}$

QUESTION 3

Find every angle on $0^\circ \leq \theta < 360^\circ$ that matches the given condition.

a) $sin\,\theta = \frac{\sqrt{3}}{2}$

b) $tan\,\theta = -1$

c) $sin\,\theta = 0$

QUESTION 4

Morgan says $cos\,150^\circ = \frac{\sqrt{3}}{2}$ because the reference angle is 30°. What part is right? What sign is missing?

QUESTION 5

Given $sin\,\theta = -\frac{5}{13}$ and θ is in quadrant IV, find exact values of $cos\,\theta$ and $tan\,\theta$.

QUESTION 6

Choose a special angle. Give four coterminal angles, two in degrees and two in radians, and show that all have the same sine and cosine.

Simple Trigonometric Equations

Name Block Date

QUESTION 1

For $sin\,\theta = \frac{1}{2}$, first find the reference angle. Then use the sign of sine to choose the quadrants.
Why are there two answers from 0° to 360°?

QUESTION 2

Solve from $0^\circ \leq \theta < 360^\circ$.

a) $cos\,\theta = \frac{1}{2}$

b) $cos\,\theta = -\frac{1}{2}$

c) $tan\,\theta = -1$

d) $cos\,\theta = 0$

QUESTION 3

Solve from $0 \leq \theta < 2\pi$.

a) $sin\,\theta = \frac{\sqrt{2}}{2}$

b) $tan\,\theta = \sqrt{3}$

c) $sin\,\theta = 0$

QUESTION 4

Sasha solves $2\,cos\,\theta - 1 = 0$ and gives only $60°$. Check that answer, then find what is missing on $0° \leq \theta < 360°$.

QUESTION 5

Solve $3\,sin\,\theta + 1 = 0$ for $0^\circ \leq \theta < 360^\circ$. Give answers to the nearest tenth of a degree.

QUESTION 6

Write a simple trig equation with exactly two solutions on $0^\circ \leq \theta < 360^\circ$. Give the solutions and explain how you know there are no others.

Angles and Circular Motion Word Problems

Name Block Date

QUESTION 1

A Ferris wheel has radius 12 m. A rider travels through an angle of $\frac{5\pi}{6}$ radians. How far does the rider move along the circle?

What do you know?

What are you finding?

Which units fit the formula?

Write the relationship

QUESTION 2

A bicycle wheel has radius 0.35 m. How many complete turns does it make over 1.1 km? Give a sensible whole-number estimate.

QUESTION 3

A sprinkler rotates from 25° to 230°. Its water reaches 8 m. Find the area watered during the turn.

QUESTION 4

The tip of a 9 cm clock hand moves 12 cm along its circular path. Find the angle turned in radians and degrees.

QUESTION 5

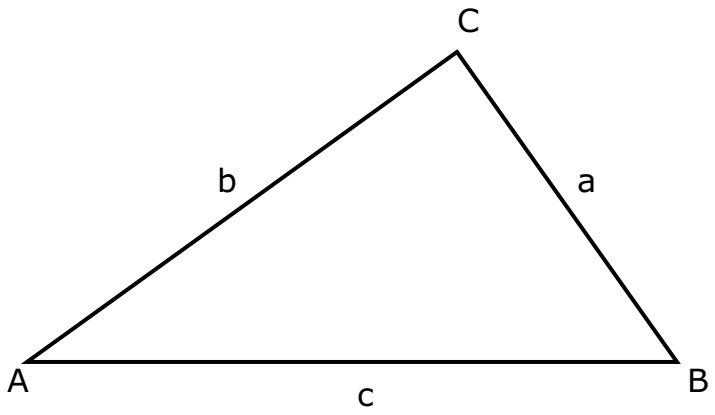
A wheel turns at 90 revolutions per minute. Find its angular speed in radians per second. Then find the linear speed of a point 0.20 m from the centre.

Sine Law

Name Block Date

QUESTION 1

In the triangle, each side is opposite the angle with the same capital letter. Write the three opposite side-angle pairs. Why does keeping those pairs straight matter?



QUESTION 2

Find the missing value. Give lengths to the nearest tenth and angles to the nearest degree.

a) $A = 42°, a = 12cm, B = 68°$

b) $A = 35°, a = 7m, b = 9m$

c) $B = 81°, b = 15km, a = 11km$

QUESTION 3

For $A = 35°, a = 7, b = 9$, explain why a calculator gives one angle for *B* but the triangle may have two. Find both possible angles and decide whether each makes a triangle.

QUESTION 4

Solve each triangle. State if there are zero, one, or two possible triangles.

a) $A = 60°, a = 12, b = 9$

b) $A = 25°, a = 5, b = 13$

c) $A = 40°, a = 7, b = 8$

QUESTION 5

Lee writes $\frac{a}{\sin B} = \frac{b}{\sin A}$. Use the labelled triangle to explain what does not match, then correct the equation.

QUESTION 6

Create an SSA set of measurements that gives two possible triangles. Show both angle possibilities and explain why both fit.

Cosine Law and Choosing a Method

Name Block Date

QUESTION 1

For each set of information, choose sine law, cosine law, or ordinary right-triangle trig. Give one reason. Do not solve.

a) two sides and their included angle

b) two angles and one side

c) three sides

d) a right triangle with one acute angle and the hypotenuse

QUESTION 2

Use the cosine law. Give lengths to the nearest tenth and angles to the nearest degree.

a) $a = 8, b = 13, C = 50^\circ$

b) $a = 7, b = 9, c = 11.6$

c) $a = 12, c = 18, B = 62^\circ$

QUESTION 3

Solve the triangle with sides $a = 7, b = 10, c = 13$. Use a second relationship to check your angles.

QUESTION 4

Taylor uses $c^2 = a^2 + b^2$ for a triangle with $a = 6, b = 9, C = 70^\circ$. What assumption is hidden in that equation? Find *c* correctly.

QUESTION 5

A triangle has sides 5 cm, 7 cm, and 11 cm. Find its largest angle. Explain how you knew which angle to find.

QUESTION 6

Two triangles each have sides of 8 units and 12 units around the included angle. One included angle is 40°; the other is 120°. Find the third side in each triangle. Which angle gives the longer third side?

Non-right-triangle Word Problems

Name Block Date

QUESTION 1

Two trails leave a campsite at an angle of $68°$. One hiker walks 4.2 km on one trail and another walks 6.1 km on the other. How far apart are they?

What do you know?

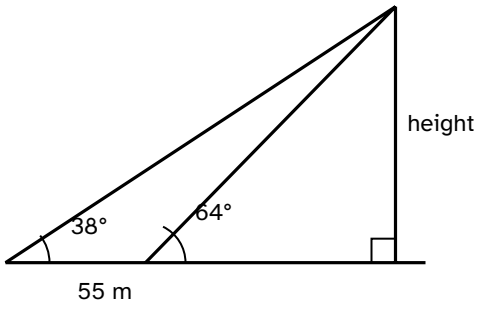
What are you finding?

Sketch and label

Which law fits?

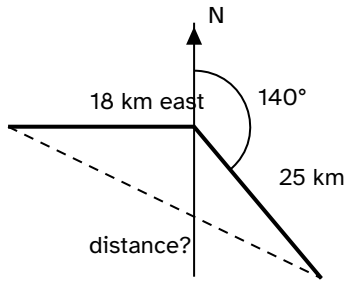
QUESTION 2

From a point on level ground, the angle of elevation to the top of a tower is $38°$. From a second point 55 m closer to the tower, the angle is $64°$. Find the tower height to the nearest metre.



QUESTION 3

A boat travels 18 km east, then travels 25 km on a bearing of $140°$.
How far is it from its starting point?



QUESTION 4

A surveyor sees two landmarks from point P. One is 320 m away and the other is 470 m away.
The angle between the sight lines is $52°$. Find the distance between the landmarks.

QUESTION 5

Two rescue stations A and B are 24 km apart. A distress signal is seen at 58° from the line AB at A and 47° from the line BA at B. Find the distance from each station to the signal.

Compound Interest and Financial Growth

Name Block Date

QUESTION 1

A $1,000 investment earns 5% once per year. Predict whether the interest earned in year 2 will be less than, equal to, or greater than the interest earned in year 1. Then calculate both amounts.

QUESTION 2

Find the future value. Round money to the nearest cent.

a) $2,000 at 4% compounded yearly for 5 years

b) $1,500 at 6% compounded monthly for 3 years

c) $8,000 at 4.5% compounded quarterly for 6 years

QUESTION 3

Find the original amount needed.

a) to have $5,000 after 8 years at 3% yearly

b) to have $12,000 after 5 years at 5.2% monthly

QUESTION 4

Two accounts advertise 6%. Account A compounds yearly. Account B compounds monthly.
Compare the balance after 4 years on a $3,000 deposit. Explain the difference.

QUESTION 5

Find the time needed for $4,000 to grow to $6,000 at 5% compounded yearly. Give the answer to the nearest tenth of a year.

QUESTION 6

Create two investments with the same starting amount and rate but different compounding periods. Predict which grows more, then compare after 10 years.

Financial Decision Word Problems

Name Block Date

QUESTION 1

A student puts $150 at the end of every month into an account earning 4.8% compounded monthly. Use technology to find the balance after 3 years. Compare the deposits with the interest earned.

Payment amount

Number of payments

Rate each payment period

What will you compare?

QUESTION 2

Plan A charges $38 per month plus $0.08 per text after 500 texts. Plan B charges $52 per month with unlimited texts. At what monthly usage do the plans cost the same? Which plan is cheaper on each side of that point?

QUESTION 3

A laptop costs $1,600 cash. A payment plan asks for $175 at the end of each month for 10 months. The account used for the cash purchase could earn 4% per year. Compare the choices in today's dollars and name one other factor that matters.

QUESTION 4

A car can be leased for $390 per month for 36 months with $2,000 due at signing and a $17,000 buyout. Or it can be bought now for $29,500. Compare total cash paid if the driver keeps the car after three years. State two important facts the totals leave out.

QUESTION 5

Jordan can invest $4,000 at 5.4% compounded monthly or use it to pay down a credit-card balance charging 19.9% compounded monthly. Compare the one-year dollar change in each choice, assuming no new purchases or payments.