The Real Number System (Key)

Name Block Date

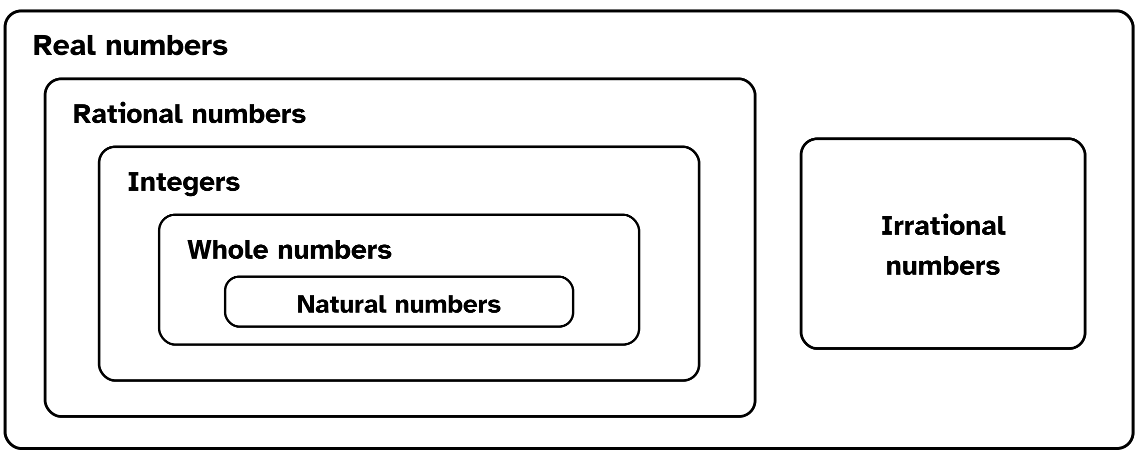
QUESTION 1

Put these numbers into two groups in any way that makes sense to you. Give each group a name. A number can belong to both groups.

-6 $\quad 0 \quad$ $\frac{5}{8}$ $\quad \sqrt{49} \quad$ $\sqrt{2}$ $\quad \pi$

QUESTION 2

The boxes show how the main sets of real numbers fit together. For each number below, write the smallest box it belongs in.



-12 $\quad 0 \quad$ 9 $\quad -\frac{4}{5}$

$\sqrt{81}$ $\quad \sqrt{7} \quad$ 3.14159 $\quad 0.1010010001\ldots$

-12: integer

0: whole number

9: natural number

$-\frac{4}{5}$: rational number

$\sqrt{81}$: natural number

$\sqrt{7}$: irrational number

3.14159: rational number

$0.1010010001\ldots$: irrational number

QUESTION 3

For each decimal, decide whether it must be rational, must be irrational, or cannot be decided from the information given. Say what made you decide.

a) The decimal stops after 17 digits.

Rational. Every terminating decimal can be written as a fraction.

b) After a few digits, the block 407 repeats forever.

Rational. The decimal eventually repeats.

c) The zeros between the 1s keep increasing: $0.101001000100001\ldots$

Irrational. The decimal never ends and no fixed block repeats.

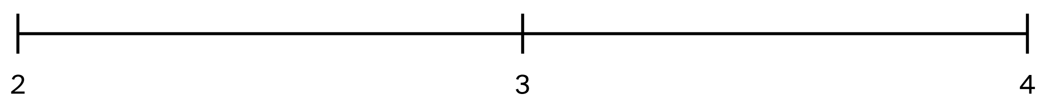
d) A calculator screen shows a decimal that stops at its last display position.

Not enough information. The calculator may have rounded or cut off more digits.

QUESTION 4

Put these numbers in order from least to greatest without changing them to decimals. Use nearby perfect squares to back up your order.

$\sqrt{5}$ $\qquad$ $\sqrt{8}$ $\qquad$ 3 $\qquad$ $\sqrt{10}$ $\qquad$ $\sqrt{12}$



$\sqrt{5} < \sqrt{8} < 3 < \sqrt{10} < \sqrt{12}$

Since $4 < 5 < 8 < 9$, both $\sqrt{5}$ and $\sqrt{8}$ lie between 2 and 3. Since $9 < 10 < 12 < 16$, $\sqrt{10}$ and $\sqrt{12}$ lie between 3 and 4.

QUESTION 5

Dev says, "$\sqrt{12}$ and $\sqrt{3}$ are irrational, so their quotient must be irrational." Dev calculates:

$$\frac{\sqrt{12}}{\sqrt{3}} = \sqrt{\frac{12}{3}} = \sqrt{4} = 2$$

Check Dev's calculation. What does it show about his claim? Then write a claim about irrational numbers that is actually true.

Dev's calculation is correct, so his claim is false. Two irrational numbers can have a rational quotient. One true claim is that an irrational number cannot be written as a ratio of two integers.

QUESTION 6

Create two irrational numbers whose sum is rational. Then create two irrational numbers whose sum is irrational. Show enough work to check both examples.

Rational Exponents (Key)

Name Block Date

QUESTION 1

Find each value without a calculator.

a) $\sqrt{25}$

$\sqrt{25} = 5$

b) $\sqrt[3]{8}$

$\sqrt[3]{8} = 2$

c) $\sqrt[4]{16}$

$\sqrt[4]{16} = 2$

QUESTION 2

Switch between radical notation and rational exponents. Assume the variables are positive.

a) $x^{3/2}$

$(\sqrt{x})^3$ or $\sqrt{x^3}$

b) $y^{2/3}$

$(\sqrt[3]{y})^2$ or $\sqrt[3]{y^2}$

c) $\sqrt[5]{p^3}$

$p^{3/5}$

d) $(\sqrt[4]{q})^7$

$q^{7/4}$

e) $\dfrac{1}{\sqrt[3]{a^2}}$

$a^{-2/3}$

QUESTION 3

Evaluate without a calculator. Write enough work to show what the numerator, denominator, and negative sign are doing.

a) $27^{2/3}$

$(\sqrt[3]{27})^2 = 3^2 = 9$

b) $16^{3/4}$

$(\sqrt[4]{16})^3 = 2^3 = 8$

c) $32^{2/5}$

$(\sqrt[5]{32})^2 = 2^2 = 4$

d) $81^{-3/4}$

$$\frac{1}{(\sqrt[4]{81})^3} = \frac{1}{27}$$

e) $125^{-2/3}$

$$\frac{1}{(\sqrt[3]{125})^2} = \frac{1}{25}$$

f) $\left(\frac{64}{125}\right)^{2/3}$

$$\left(\frac{4}{5}\right)^2 = \frac{16}{25}$$

QUESTION 4

For each expression, choose whether you would take the root first or the power first. Evaluate it, then explain which route kept the numbers easier to handle.

a) $64^{2/3}$

Root first: $(\sqrt[3]{64})^2 = 4^2 = 16$.

b) $16^{3/2}$

Root first: $(\sqrt{16})^3 = 4^3 = 64$.

c) $81^{3/4}$

Root first: $(\sqrt[4]{81})^3 = 3^3 = 27$. Taking the root first avoids large powers.

QUESTION 5

Sienna writes:

> $16^{-3/4} = -\left(\sqrt[4]{16}\right)^3 = -8$

Her root and power are correct, but her final answer is not. Explain what the negative exponent means, correct her work, and check your answer using $16^{3/4}$.

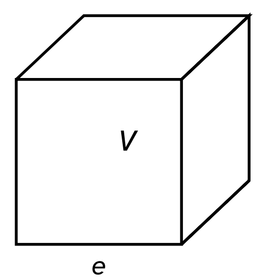
A negative exponent means take the reciprocal. It does not make the value negative.

$$16^{-3/4} = \frac{1}{16^{3/4}} = \frac{1}{(\sqrt[4]{16})^3} = \frac{1}{8}$$

Check: $\frac{1}{8} \cdot 16^{3/4} = \frac{1}{8} \cdot 8 = 1$.

QUESTION 6

A cube has volume *V* and edge length *e*. Write the edge length and total surface area using powers of *V*. Then find both measurements when $V = 216$ cm^3. Finally, explain what happens to the edge length and surface area if the volume is multiplied by 27.



$e = V^{1/3}$ and $SA = 6e^2 = 6V^{2/3}$.

When $V = 216$ cm^3, $e = 6$ cm and $SA = 6(6^2) = 216$ cm^2.

If volume is multiplied by 27, edge length is multiplied by $27^{1/3} = 3$, and surface area is multiplied by $27^{2/3} = 9$.

Simplifying Radicals (Key)

Name Block Date

QUESTION 1

A square has area 72 cm^2, so its exact side length is $\sqrt{72}$ cm. Noor says the same length can be written as $6\sqrt{2}$ cm.

Square $6\sqrt{2}$ to check Noor's claim. Then use $72 = 36 \cdot 2$ to show how $\sqrt{72}$ becomes $6\sqrt{2}$ without using decimals.

$(6\sqrt{2})^2 = 36 \cdot 2 = 72$, so $6\sqrt{2}$ is the positive side length.

$\sqrt{72} = \sqrt{36 \cdot 2} = \sqrt{36}\sqrt{2} = 6\sqrt{2}$.

QUESTION 2

Rewrite the number under each radical using a useful square, cube, or fourth power. Then simplify.

a) $\sqrt{48}$

$\sqrt{16 \cdot 3} = 4\sqrt{3}$

b) $\sqrt{75}$

$\sqrt{25 \cdot 3} = 5\sqrt{3}$

c) $\sqrt{180}$

$\sqrt{36 \cdot 5} = 6\sqrt{5}$

d) $\sqrt[3]{54}$

$\sqrt[3]{27 \cdot 2} = 3\sqrt[3]{2}$

e) $\sqrt[4]{162}$

$\sqrt[4]{81 \cdot 2} = 3\sqrt[4]{2}$

QUESTION 3

Simplify. Assume all variables are positive. In each answer, make sure nothing else can come out of the radical.

a) $\sqrt{72x^5}$

$\sqrt{36x^4 \cdot 2x} = 6x^2\sqrt{2x}$

b) $\sqrt[3]{54a^7}$

$\sqrt[3]{27a^6 \cdot 2a} = 3a^2\sqrt[3]{2a}$

c) $\sqrt[4]{80b^9}$

$\sqrt[4]{16b^8 \cdot 5b} = 2b^2\sqrt[4]{5b}$

QUESTION 4

Mateo writes:

> $\sqrt{50} = \sqrt{25+25} = \sqrt{25} + \sqrt{25} = 10$

Explain why the middle step can look reasonable even though it is not allowed. Fix the simplification, then square both Mateo's answer and your answer to check which one works.

Mateo used the true fact $50 = 25 + 25$, but square roots do not split across addition. The multiplication rule may have made this look reasonable.

$\sqrt{50} = \sqrt{25 \cdot 2} = 5\sqrt{2}$. Checks: $10^2 = 100$, but $(5\sqrt{2})^2 = 25 \cdot 2 = 50$.

QUESTION 5

Decide whether each statement is true. Back up each decision without using decimals.

a) $3\sqrt{5} = \sqrt{45}$

True: $\sqrt{45} = \sqrt{9 \cdot 5} = 3\sqrt{5}$.

b) $2\sqrt{7} = \sqrt{14}$

False: $2\sqrt{7} = \sqrt{4 \cdot 7} = \sqrt{28}$.

c) $4\sqrt{3} = \sqrt{48}$

True: $\sqrt{48} = \sqrt{16 \cdot 3} = 4\sqrt{3}$.

d) $2\sqrt[3]{5} = \sqrt[3]{40}$

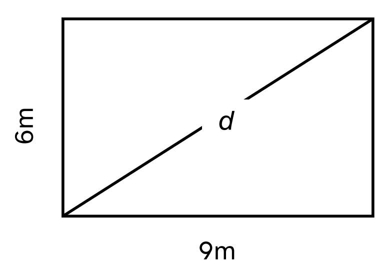
True: $\sqrt[3]{40} = \sqrt[3]{8 \cdot 5} = 2\sqrt[3]{5}$.

Use the pattern to write a rule for moving a positive number *k* inside an *n*th root.

$k\sqrt[n]{a} = \sqrt[n]{k^n a}$ for positive k.

QUESTION 6

A rectangular room measures 9 m by 6 m. Find the exact diagonal in simplest radical form. Show that it lies between 10 m and 11 m without using a calculator. A similar room has every length multiplied by a positive scale factor *k*. Find its diagonal and explain what the result says about scaled lengths.



$d = \sqrt{9^2 + 6^2} = \sqrt{117} = 3\sqrt{13}$ m.

Since $100 < 117 < 121$, $10 < \sqrt{117} < 11$.

The new diagonal is $\sqrt{(9k)^2 + (6k)^2} = 3k\sqrt{13}$ m, so every length is multiplied by k.

Operations with Radicals (Key)

Name Block Date

QUESTION 1

Compare $2\sqrt{3} + 5\sqrt{3}$ with $2\sqrt{3} + 5\sqrt{2}$. Explain why the first one combines to $7\sqrt{3}$ but the second one does not. What needs to match before radical terms can be combined?

In the first expression, both terms have the same radical part, $\sqrt{3}$, so their coefficients add: $2 + 5 = 7$. In the second expression, $\sqrt{3}$ and $\sqrt{2}$ are different. Radical terms combine only when their simplified radical parts match.

QUESTION 2

Simplify first when needed. Then combine like radical terms.

a) $\sqrt{12} + \sqrt{27}$

$2\sqrt{3} + 3\sqrt{3} = 5\sqrt{3}$

b) $3\sqrt{20} - \sqrt{45}$

$6\sqrt{5} - 3\sqrt{5} = 3\sqrt{5}$

c) $2\sqrt{18} + \sqrt{8} - \sqrt{50}$

$6\sqrt{2} + 2\sqrt{2} - 5\sqrt{2} = 3\sqrt{2}$

d) $2\sqrt[3]{16} - \sqrt[3]{54}$

$4\sqrt[3]{2} - 3\sqrt[3]{2} = \sqrt[3]{2}$

QUESTION 3

Before calculating, predict whether each answer will be rational or irrational. Then multiply and simplify.

a) $\sqrt{6} \cdot \sqrt{15}$

Irrational: $\sqrt{90} = 3\sqrt{10}$.

b) $(2\sqrt{3})(5\sqrt{6})$

Irrational: $10\sqrt{18} = 30\sqrt{2}$.

c) $(\sqrt{5}+2)(\sqrt{5}-2)$

Rational: $5 - 4 = 1$.

d) $(3+\sqrt{2})^2$

Irrational: $9 + 6\sqrt{2} + 2 = 11 + 6\sqrt{2}$.

Why did the radical terms disappear in part c but not in part d?

Part c uses conjugates. The middle terms are equal and opposite, so they cancel. In part d the two middle products have the same sign, so they add.

QUESTION 4

Harper writes:

> $\sqrt{8}+\sqrt{18}=\sqrt{8+18}=\sqrt{26}$

Explain why Harper may have expected radicals to combine this way. Use a quick estimate to show the answer cannot be right, then fix the addition. Can $\sqrt{a}+\sqrt{b}=\sqrt{a+b}$ ever be true when both *a* and *b* are positive?

The true multiplication rule $\sqrt{a}\sqrt{b}=\sqrt{ab}$ may have made the addition look reasonable. But $\sqrt{8}+\sqrt{18}\approx 2.8+4.2=7$, while $\sqrt{26}\approx 5.1$.

$\sqrt{8}+\sqrt{18}=2\sqrt{2}+3\sqrt{2}=5\sqrt{2}$. The proposed rule cannot be true for positive a, b: squaring both sides would require $2\sqrt{ab}=0$.

QUESTION 5

Divide and simplify. Leave no radical in the denominator.

a) $\dfrac{\sqrt{72}}{\sqrt{2}}$

$\sqrt{36}=6$

b) $\dfrac{6\sqrt{15}}{3\sqrt{5}}$

$2\sqrt{3}$

c) $\dfrac{5}{\sqrt{2}}$

$\dfrac{5\sqrt{2}}{2}$

d) $\dfrac{3}{2\sqrt{3}}$

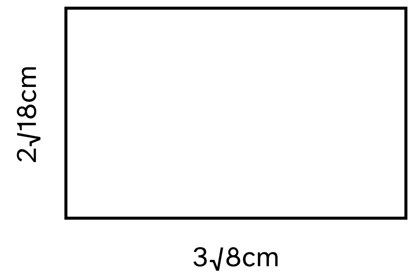
$\dfrac{3\sqrt{3}}{6}=\dfrac{\sqrt{3}}{2}$

Why does multiplying by $\dfrac{\sqrt{a}}{\sqrt{a}}$ change the form but not the value? What must be true about *a*?

The multiplier equals 1, so it does not change the value. For a real, nonzero denominator, $a>0$.

QUESTION 6

A rectangle has the exact dimensions shown. Decide whether it is actually a square. Then find its exact perimeter and area. Explain why one answer is irrational while the other is rational.



$3\sqrt{8} = 6\sqrt{2}$ cm and $2\sqrt{18} = 6\sqrt{2}$ cm, so it is a square.

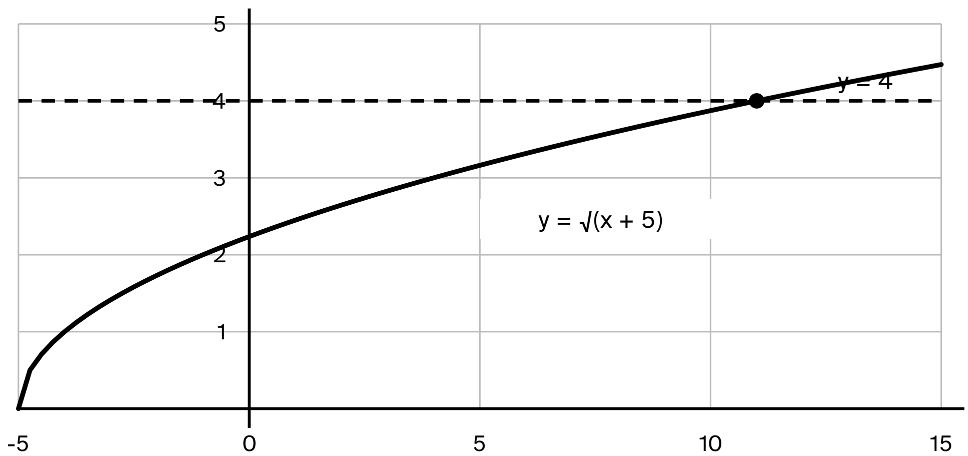
$P = 4(6\sqrt{2}) = 24\sqrt{2}$ cm and $A = (6\sqrt{2})^2 = 72$ cm^2. The perimeter keeps one factor of $\sqrt{2}$; in the area, the two radical factors multiply to 2.

Radical Equations (Key)

Name Block Date

QUESTION 1

The graph shows $y = \sqrt{x+5}$ and $y = 4$. Explain why the *x*-coordinate of their intersection solves $\sqrt{x+5} = 4$. Then solve the equation algebraically and connect your steps to the graph.



The intersection is where both expressions have the same y-value. Algebraically, $x + 5 = 16$, so $x = 11$. The graph's intersection is therefore $(11, 4)$.

QUESTION 2

Solve each equation. Isolate the radical before squaring, and check your result in the original equation.

a) $\sqrt{2x+3} = 7$

$2x+3 = 49 \Rightarrow x = 23$. Check: $\sqrt{49} = 7$.

b) $\sqrt{x-4} + 2 = 8$

$\sqrt{x-4} = 6 \Rightarrow x-4 = 36 \Rightarrow x = 40$. Check: $\sqrt{36} + 2 = 8$.

c) $3\sqrt{x+1} - 2 = 10$

$3\sqrt{x+1} = 12 \Rightarrow \sqrt{x+1} = 4 \Rightarrow x = 15$. Check: $3\sqrt{16} - 2 = 10$.

QUESTION 3

Solve $\sqrt{x+6} = x$. Before squaring, record what must be true about *x*. After solving the resulting quadratic equation, check every candidate in the original equation and explain why any rejected value appeared.

The radical requires $x \geq -6$, and its output is non-negative, so the right side requires $x \geq 0$.

$x + 6 = x^2 \Rightarrow x^2 - x - 6 = 0 \Rightarrow (x-3)(x+2) = 0$. Candidates are $x = 3, -2$.

$x = 3$ works: $\sqrt{9} = 3$. $x = -2$ fails because $\sqrt{4} = 2 \neq -2$. Squaring removed the sign difference.

QUESTION 4

Alina writes:

> $\sqrt{x+1} = x - 1 \Rightarrow x + 1 = (x-1)^2 \Rightarrow x^2 - 3x = 0 \Rightarrow x = 0, 3$
>
> She reports both values as solutions.

Check Alina's algebra and both candidates. What was missing from her solution, and why can squaring create a candidate that does not work?

Her algebra is correct, but she did not check the candidates. For $x = 0$, $\sqrt{1} = 1$ but $x - 1 = -1$, so it fails. For $x = 3$, both sides equal 2. The solution is $x = 3$. Squaring makes positive and negative values look the same.

QUESTION 5

Decide whether each claim is always, sometimes, or never true. Give a reason or a counterexample.

a) A real solution makes every square-root radicand non-negative.

Always. Otherwise the original equation is not defined over the real numbers.

b) Every solution found after squaring solves the original equation.

Sometimes. Squaring can introduce an extraneous candidate.

c) An equation of the form $\sqrt{ax+b}=c$, with $c<0$, has a real solution.

Never. A principal square root cannot be negative.

d) Checking candidates may remove candidates, but it cannot create new ones.

Always. Checking only tests the candidates already found.

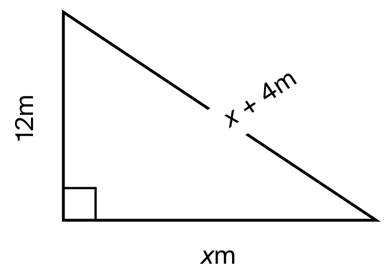
QUESTION 6

A right triangular support has the dimensions shown.

a) Write a radical equation for *x* and predict whether *x* is shorter or longer than 12 m.

b) Solve the equation and find all three side lengths.

c) Check the measurements using the Pythagorean theorem.



a) $\sqrt{x^2+144}=x+4$. The diagram suggests $x>12$.

b) $x^2+144=(x+4)^2=x^2+8x+16$, so $x=16$. The sides are 12 m, 16 m, and 20 m.

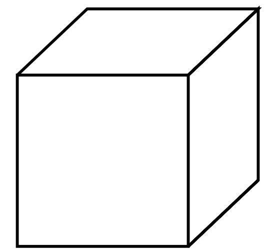
c) $12^2+16^2=144+256=400=20^2$.

Powers and Radicals Word Problems (Key)

Name Block Date

QUESTION 1

A cube-shaped storage bin holds 512 L. One litre is 1 dm^3. The company needs the edge length and the total outside area before ordering material.



Organize the problem before solving it.

What we know	$V = 512$ dm^3
What we need	edge length e and total surface area
Useful relationships	$V = e^3$ and $SA = 6e^2$
Units to report	dm and dm^2

Now find both measurements and check that they make sense for the stated volume.

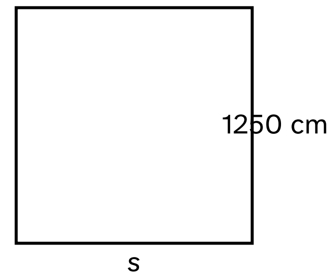
$e = 512^{1/3} = 8$ dm.

$SA = 6(8^2) = 384$ dm^2.

Check: $8^3 = 512$, so the edge length gives the correct volume.

QUESTION 2

A square mosaic has area 1250 cm^2. A metal strip will run around its entire edge. Find the exact side length and exact strip length. Then give the strip length to the nearest tenth of a centimetre.

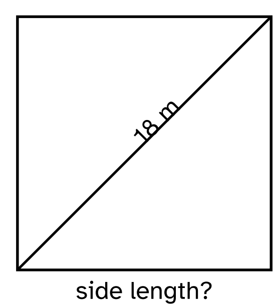


$s = \sqrt{1250} = \sqrt{625 \cdot 2} = 25\sqrt{2}$ cm.

The strip length is the perimeter: $4s = 100\sqrt{2} \approx 141.4$ cm.

QUESTION 3

A square courtyard has a diagonal of 18 m. Find the exact side length and the perimeter. Then give the perimeter to the nearest tenth of a metre.



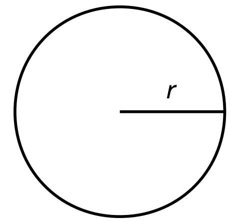
Let the side length be s. The diagonal makes a right triangle.

$s^2 + s^2 = 18^2$
$2s^2 = 324$
$s^2 = 162$
$s = 9\sqrt{2}$ m

The perimeter is $4s = 36\sqrt{2}$ **m, or about 50.9 m**.

QUESTION 4

A circular community garden has area 450π m^2. Fencing will go around the whole garden. Find the exact radius and exact fence length. Then give the fence length to the nearest tenth of a metre.



$\pi r^2 = 450\pi \Rightarrow r = \sqrt{450} = 15\sqrt{2}$ m.

$C = 2\pi r = 30\pi\sqrt{2} \approx 133.3$ m.

QUESTION 5

Two cube-shaped shipping boxes are similar. The larger box has 64 times the volume of the smaller box. The smaller box has an edge length of 12 cm. Find the larger edge length. Then compare the amount of cardboard needed for the two closed boxes by finding the ratio of their surface areas. Explain why the surface-area ratio is not 64.

The edge-length factor is $64^{1/3} = 4$, so the larger edge is $4(12) = 48$ cm.

The surface-area factor is $4^2 = 16$. The volume factor uses three dimensions, but surface area uses two, so its factor is 16 rather than 64.

Common Factors and Structure (Key)

Name Block Date

QUESTION 1

Expand $6x(2x+3)$. Then factor $12x^2+18x$. Explain how your two calculations undo each other.

$6x(2x+3)=12x^2+18x$.

$12x^2+18x=6x(2x+3)$. Expanding distributes the common factor into both terms; factoring pulls that same factor back out.

QUESTION 2

Factor each polynomial by taking out its greatest common factor. Expand your answer for one of them as a check.

a) $8x+12$

$4(2x+3)$

b) $15a^2-20a$

$5a(3a-4)$

c) $18m^3n+24m^2n^2$

$6m^2n(3m+4n)$

d) $14p^3-21p^2+7p$

$7p(2p^2-3p+1)$

QUESTION 3

Factor completely. If the first term is negative, decide whether taking out a negative factor makes the result easier to read.

a) $-14x^2 + 21x$

$-7x(2x-3)$

b) $9r^4s^2 - 12r^2s^3$

$3r^2s^2(3r^2-4s)$

c) $-10y^3 - 25y^2 + 15y$

$-5y(2y^2+5y-3)$

d) $16u^2v - 24uv^2 + 8uv$

$8uv(2u-3v+1)$

QUESTION 4

Leah and Owen factor the same polynomial.

Leah

$12x^2 + 18x = 3x(4x+6)$

Owen

$12x^2 + 18x = 6x(2x+3)$

Expand both answers. Explain why both are equivalent to the original polynomial, but only one has removed the greatest common factor. How can you tell whether a common factor remains?

Both expand to $12x^2 + 18x$. Leah removed a common factor, but 2 still divides both terms inside her brackets. Owen removed the greatest common factor, $6x$. Check the terms inside the brackets: they should have no common factor other than 1.

QUESTION 5

Decide whether each polynomial has been factored completely using common factors. If it has not, finish the job.

a) $6x^2 + 9x = 3x(2x + 3)$

Complete. No common factor remains.

b) $20a^2 - 30a = 5a(4a - 6)$

Not complete: $10a(2a - 3)$.

c) $-12m^3 - 18m^2 = -6m^2(2m + 3)$

Complete. No common factor remains.

d) $14p^3 - 21p^2 + 35p = 7p(2p^2 - 3p + 5)$

Complete using common factors.

QUESTION 6

A concert has $15x^2 + 25x$ seats arranged in $5x$ equal rows. Factor the total to find the number of seats in each row. When $x = 4$, find the number of rows, the seats in each row, and the total number of seats. Multiply to check.

$15x^2 + 25x = 5x(3x + 5)$, so each row has $3x + 5$ seats.

When $x = 4$, there are 20 rows with 17 seats in each row, for **340 seats**.

Check: $20(17) = 340$ and $5x(3x + 5) = 15x^2 + 25x$.

Factoring Polynomials Starting with x Squared (Key)

Name Block Date

QUESTION 1

Expand each product. Keep the two middle products separate until the last line.

a) $(x+2)(x+5)$

$x^2+5x+2x+10=x^2+7x+10$

b) $(x-3)(x-4)$

$x^2-4x-3x+12=x^2-7x+12$

c) $(x+6)(x-2)$

$x^2-2x+6x-12=x^2+4x-12$

For each result, compare the two numbers in the factors with the middle coefficient and the constant term. What two calculations keep appearing?

The two numbers add to the middle coefficient and multiply to the constant term.

QUESTION 2

Factor each polynomial. Expand every answer to check both the middle term and the constant term.

a) $x^2 + 9x + 20$

$(x + 4)(x + 5)$

Check: $x^2 + 5x + 4x + 20$.

b) $x^2 + 11x + 24$

$(x + 3)(x + 8)$

Check: $x^2 + 8x + 3x + 24$.

c) $x^2 - 9x + 20$

$(x - 4)(x - 5)$

Check: $x^2 - 5x - 4x + 20$.

d) $x^2 + x - 20$

$(x + 5)(x - 4)$

Check: $x^2 - 4x + 5x - 20$.

QUESTION 3

Factor completely.

a) $x^2 + 7x + 10$

$(x+5)(x+2)$

b) $x^2 + 13x + 36$

$(x+4)(x+9)$

c) $x^2 - 2x - 35$

$(x-7)(x+5)$

d) $x^2 - 12x + 36$

$(x-6)^2$

e) $x^2 - 5x - 24$

$(x-8)(x+3)$

f) $x^2 + 2x - 48$

$(x+8)(x-6)$

QUESTION 4

Predict the signs in the factors before you work out the numbers. Then factor.

a) $x^2 + 8x + 15$

$(x+3)(x+5)$

b) $x^2 - 8x + 15$

$(x-3)(x-5)$

c) $x^2 + 2x - 15$

$(x+5)(x-3)$

d) $x^2 - 2x - 15$

$(x+3)(x-5)$

QUESTION 5

Mira writes:

> $x^2 + 7x + 12 = (x+2)(x+6)$
>
> “Two times six is twelve, so the factors work.”

Expand Mira's factors and check her work. Explain what she forgot to check, then write and verify the correct factorization.

$(x+2)(x+6) = x^2 + 8x + 12$, so the middle term does not match. Mira checked the product, but not the sum.

The correct factorization is $(x+3)(x+4)$. Check: $x^2 + 4x + 3x + 12 = x^2 + 7x + 12$.

QUESTION 6

A rectangular mural has area $x^2 + 5x - 24$ m^2. One side is $x + 8$ m. Find the other side and the values of *x* that make both side lengths positive. Then write the perimeter in simplest form and use expansion to check the area.

$x^2 + 5x - 24 = (x+8)(x-3)$, so the other side is $x - 3$ m.

Both sides are positive when $x > 3$. The perimeter is $2[(x+8) + (x-3)] = 4x + 10$ m.

Check: $(x+8)(x-3) = x^2 + 5x - 24$.

Factoring Polynomials (Key)

Name Block Date

QUESTION 1

Jules thinks that since 3 and 4 multiply to 12, this should work:

$$2x^2 + 11x + 12 = (x+3)(x+4)$$

a) Expand out Jules' factors and check his work. What doesn't match the original?

$(x+3)(x+4) = x^2 + 4x + 3x + 12 = \ x^2 + 7x + 12$

The first number should be **2**, not 1. The middle number should be **11**, not 7.

b) What did Jules forget to account for?

Jules forgot about the **2 in front of** x^2.

QUESTION 2

Factor $2x^2 + 11x + 12$. Show how you split the middle term, then group. Multiply your factors to check.

$(2)(12) = 24$, so use 3 and 8 because $3 + 8 = 11$ and $3 \cdot 8 = 24$.

$2x^2 + 3x + 8x + 12 = x(2x+3) + 4(2x+3)$

$(x+4)(2x+3)$

Check: $(x+4)(2x+3) = 2x^2 + 3x + 8x + 12 = 2x^2 + 11x + 12$.

QUESTION 3

Factor each polynomial. Multiply your factors to check.

a) $3x^2 + 10x + 8$

$3x^2 + 6x + 4x + 8 = 3x(x + 2) + 4(x + 2)$

$(3x + 4)(x + 2)$

Check: $3x^2 + 6x + 4x + 8 = 3x^2 + 10x + 8.$

b) $2x^2 - x - 6$

$2x^2 + 3x - 4x - 6 = x(2x + 3) - 2(2x + 3)$

$(x - 2)(2x + 3)$

Check: $2x^2 + 3x - 4x - 6 = 2x^2 - x - 6.$

c) $6x^2 + 11x + 3$

$6x^2 + 9x + 2x + 3 = 3x(2x + 3) + 1(2x + 3)$

$(3x + 1)(2x + 3)$

Check: $6x^2 + 9x + 2x + 3 = 6x^2 + 11x + 3.$

QUESTION 3 CONTINUED

d) $4x^2 - 4x - 15$

$4x^2 + 6x - 10x - 15 = 2x(2x + 3) - 5(2x + 3)$

$(2x - 5)(2x + 3)$

Check: $4x^2 + 6x - 10x - 15 = 4x^2 - 4x - 15$.

e) $5x^2 - 13x + 6$

$5x^2 - 10x - 3x + 6 = 5x(x - 2) - 3(x - 2)$

$(5x - 3)(x - 2)$

Check: $5x^2 - 10x - 3x + 6 = 5x^2 - 13x + 6$.

f) $8x^2 + 2x - 3$

$8x^2 + 6x - 4x - 3 = 2x(4x + 3) - 1(4x + 3)$

$(2x - 1)(4x + 3)$

Check: $8x^2 + 6x - 4x - 3 = 8x^2 + 2x - 3$.

QUESTION 4

Sam factors $6x^2 + 7x + 2$:

> $6x^2 + 7x + 2 = (6x + 1)(x + 2)$
>
> Sam says, "One times two gives the constant term, so these factors work."

a) What part of Sam's answer matches the original?

The first term is $6x^2$, and the last term is **2**.

b) Multiply Sam's factors. What doesn't match?

$(6x + 1)(x + 2) = 6x^2 + 12x + x + 2 = 6x^2 + 13x + 2$

The middle term should be $7x$, not $13x$.

c) Sam only looked at the 2. What did that make him miss?

He missed how the **6 changes the two middle products**.

d) Fix Sam's answer. Multiply your factors to check.

$6x^2 + 4x + 3x + 2 = 2x(3x + 2) + 1(3x + 2)$

$(2x + 1)(3x + 2)$

Check: $6x^2 + 4x + 3x + 2 = 6x^2 + 7x + 2$.

QUESTION 5

Factor $18p^3 + 15p^2 - 12p$ as far as you can. How can you tell you're done?

$18p^3 + 15p^2 - 12p = 3p(6p^2 + 5p - 4)$

$= 3p(6p^2 + 8p - 3p - 4)$

$= 3p[2p(3p + 4) - 1(3p + 4)]$

$= 3p(3p + 4)(2p - 1)$

No factor is shared by all three parts, and the two linear factors cannot be split any further.

QUESTION 6

A fundraiser earns $2x^2 + 7x + 6$ dollars. The money is split equally among $x + 2$ teams. Factor the total to find each team's share. Then let *x* = 1 and find the number of teams, each team's share, and the total. Multiply to check.

$2x^2 + 7x + 6 = 2x^2 + 4x + 3x + 6$

$= 2x(x + 2) + 3(x + 2) = (x + 2)(2x + 3)$

Each team receives $2x + 3$ **dollars**.

When $x = 1$, there are 3 teams and each receives \$5, for a total of **\$15**.

Check: $3(5) = 15$ and $(x + 2)(2x + 3) = 2x^2 + 7x + 6$.

Difference of Squares and Choosing a Method (Key)

Name Block Date

QUESTION 1

Noor wants to calculate $47 \cdot 53$ mentally. She rewrites it as $(50-3)(50+3)$. Predict whether the result is greater or less than 50^2. Then calculate the product without long multiplication and explain which terms cancel.

The result is less than 50^2.

$(50-3)(50+3) = 50^2 + 150 - 150 - 3^2 = 2500 - 9 = 2491$. The two middle products are equal and opposite.

QUESTION 2

Factor completely.

a) $x^2 - 25$

$(x-5)(x+5)$

b) $9x^2 - 16$

$(3x-4)(3x+4)$

c) $49a^2 - 81b^2$

$(7a-9b)(7a+9b)$

d) $3x^2 - 75$

$3(x-5)(x+5)$

e) $8a^3 - 18a$

$2a(2a-3)(2a+3)$

f) $x^4 - 16$

$(x-2)(x+2)(x^2+4)$

QUESTION 3

Factor the three expressions. Then explain how you can quickly tell a repeated factor from a pair of conjugate factors.

a) $x^2 + 10x + 25$

$(x+5)^2$

b) $x^2 - 25$

$(x-5)(x+5)$

c) $9x^2 - 24x + 16$

$(3x-4)^2$

A repeated factor comes from a three-term perfect-square pattern. Conjugate factors come from two squared terms separated by subtraction.

QUESTION 4

Priya writes:

> $x^2 + 16 = (x+4)(x-4)$
>
> "Both terms are squares, so the pattern works."

Expand Priya's factors and check her claim. Explain why the sign between the squares matters, and state what can be said about factoring $x^2 + 16$ into real linear factors.

$(x+4)(x-4) = x^2 - 16$, not $x^2 + 16$. The pattern needs subtraction because the opposite middle terms cancel while the constants multiply to a negative value. $x^2 + 16$ has no real linear factors.

QUESTION 5

Factor completely. For each one, decide whether the first useful move is a common factor, a factor pair, a difference of squares, or a perfect-square pattern.

a) $12x^3 - 27x$

Common factor: $3x(2x-3)(2x+3)$.

b) $6x^2 + 7x + 2$

Factor pair: $(3x+2)(2x+1)$.

c) $x^2 - 14x + 49$

Perfect square: $(x-7)^2$.

d) $5x^2 - 45$

Common factor: $5(x-3)(x+3)$.

e) $2x^2 + 9x + 10$

Factor pair: $(2x+5)(x+2)$.

f) $4x^2 + 12x + 9$

Perfect square: $(2x+3)^2$.

QUESTION 6

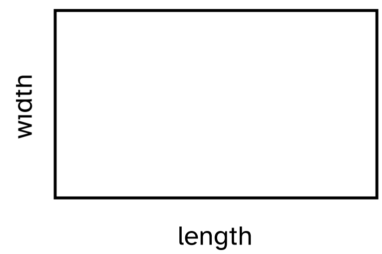
Create a polynomial that needs a common factor first and a difference of squares second. Factor it completely, then expand your answer to check it.

Factoring Word Problems (Key)

Name Block Date

QUESTION 1

A rectangular patio has area $6x^2 + 17x + 5$ m^2. Its dimensions are polynomial expressions.



Organize the problem before solving it.

What we know	area: $6x^2 + 17x + 5$ m^2
What we need	two factors that can represent length and width
Useful relationship	area = length $\times$ width
How to check	expand the factors and compare with the given area

Factor the area to find possible dimensions. When $x = 3$, find the dimensions, area, and perimeter.

$6x^2 + 17x + 5 = (3x + 1)(2x + 5)$, so possible dimensions are $3x + 1$ m and $2x + 5$ m.

When $x = 3$, the patio is 10 m by 11 m. Its area is 110 m^2 and its perimeter is 42 m.

Check: $(3x + 1)(2x + 5) = 6x^2 + 17x + 5$.

QUESTION 2

A school orders $2x^2 + 11x + 5$ badges and shares them equally among $2x + 1$ classes. Factor the total to find how many badges each class receives. When $x = 4$, find the number of classes, the badges per class, and the total. Multiply to check.

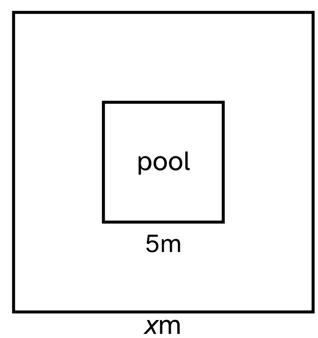
$2x^2 + 11x + 5 = (2x + 1)(x + 5)$, so each class receives $x + 5$ badges.

When $x = 4$, there are 9 classes and each receives 9 badges, for **81 badges**.

Check: $9(9) = 81$ and $2(4^2) + 11(4) + 5 = 81$.

QUESTION 3

A square courtyard has side length *x* metres. A centred square pool has side length 5 m. The remaining courtyard area is 144 m^2. Find the courtyard side length and the uniform width of the space around the pool.



$x^2 - 25 = 144$, so $x^2 = 169$. A length must be positive, so $x = 13$ m.

The uniform width is $\frac{13-5}{2} = 4$ m.

QUESTION 4

Two consecutive positive locker numbers have a product of 156. Find the two locker numbers. Show how factoring lets you reject the pair that does not fit the situation.

Let the smaller number be n. Then $n(n+1)=156$, so $n^2+n-156=0$.

$(n-12)(n+13)=0$, giving $n=12$ or $n=-13$. The numbers must be positive, so the locker numbers are 12 and 13.

QUESTION 5

A rectangular stage has area 120 m^2. Its length is 2 m greater than its width. Find both dimensions and check the area. Explain why the negative solution from the factored equation does not make sense here.

Let the width be w m. Then $w(w+2)=120$, so $w^2+2w-120=0$.

$(w+12)(w-10)=0$, so $w=-12$ or $w=10$. A width cannot be negative. The stage is 10 m by 12 m, and $10(12)=120$.

Recognizing Quadratic Relationships (Key)

Name Block Date

QUESTION 1

The three columns show different relationships. Before calculating differences, write down anything you notice. Then classify each relationship as linear, quadratic, or neither.

x	**A**	**B**	**C**
-2	1	4	2
-1	3	1	4
0	5	0	8
1	7	1	16
2	9	4	32

A is linear because its first differences are 2. B is quadratic because its first differences are $-3, -1, 1, 3$ and its second differences are 2. C is neither linear nor quadratic; its values double.

QUESTION 2

Use differences to decide whether this table is quadratic. Predict the value when $x = 5$, and explain how the differences support your answer.

x	**0**	**1**	**2**	**3**	**4**	**5**
***f*(*x*)**	3	6	11	18	27	
first difference						
second difference						

First differences: $3, 5, 7, 9$. Second differences: $2, 2, 2$, so the relationship is quadratic. The next first difference is 11, giving $f(5) = 38$.

QUESTION 3

Decide which equations define quadratic functions. For each quadratic, state whether it opens up or down.

a) $y = 3x - 2$

Not quadratic.

b) $y = -2x^2 + 5x + 1$

Quadratic; opens down.

c) $y = 2^x$

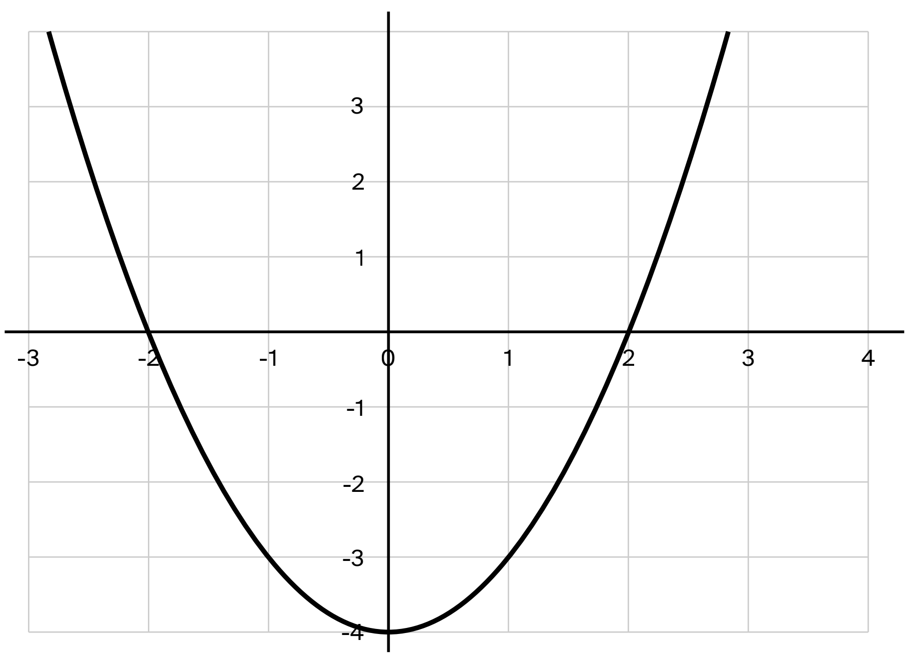
Not quadratic.

d) $y = (x-3)(x+1)$

Quadratic; opens up.

QUESTION 4

Read the graph. Find the vertex, axis of symmetry, *x*-intercepts, *y*-intercept, domain, and range.



Vertex: $(0, -4)$; axis: $x = 0$; *x*-intercepts: $(-2, 0)$ and $(2, 0)$; *y*-intercept: $(0, -4)$; domain: **all real numbers**; range: $y \geq -4$.

QUESTION 5

Lia looks at the values $4, 6, 10, 16, 24$ for equally spaced *x*-values. She says, “The first differences are not equal, so this cannot be quadratic.”

Check her conclusion. What did she overlook, and what should the next value be if the pattern continues?

The first differences are $2, 4, 6, 8$. Their differences are all 2, so the relationship is quadratic. Lia stopped one level too soon. The next first difference is 10, so the next value is 34.

QUESTION 6

For $x = -2, -1, 0, 1, 2$, a relationship has values $-3, 3, 5, 3, -3$. Find the first and second differences. What does the sign of the second differences tell you? Sketch the graph.

First differences: $6, 2, -2, -6$. Second differences: $-4, -4, -4$.

The constant negative second difference shows a quadratic that opens down. Plotting the five points gives a parabola with vertex $(0, 5)$.

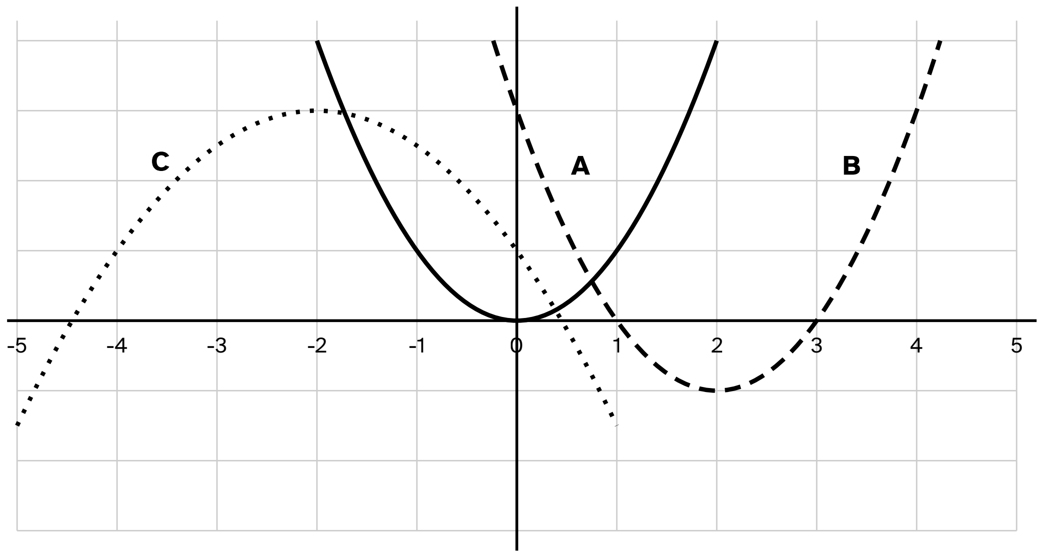
Transformations and Vertex Form (Key)

Name Block Date

QUESTION 1

Match curves A, B, and C with the equations below. For each curve, state its vertex and whether it opens up or down.

$y = x^2$ $\quad$ $y = (x-2)^2 - 1$ $\quad$ $y = -\frac{1}{2}(x+2)^2 + 3$



A: $y = x^2$, vertex $(0,0)$, opens up. B: $y = (x-2)^2 - 1$, vertex $(2,-1)$, opens up. C: $y = -\frac{1}{2}(x+2)^2 + 3$, vertex $(-2,3)$, opens down.

QUESTION 2

Describe how each graph is changed from $y = x^2$.

a) $y = 2(x-3)^2 + 1$

Right 3, up 1, vertical stretch by 2.

b) $y = -(x+4)^2$

Left 4, reflected across the *x*-axis.

c) $y = \frac{1}{3}(x-2)^2 - 5$

Right 2, down 5, vertical compression by $\frac{1}{3}$.

d) $y = -3x^2 + 2$

Up 2, reflected, vertical stretch by 3.

QUESTION 3

Write each equation in vertex form using the vertex and one other point.

a) vertex $(2, -3)$, point $(4, 5)$

$5 = a(4-2)^2 - 3 \Rightarrow a = 2$, so $y = 2(x-2)^2 - 3$.

b) vertex $(-1, 4)$, point $(1, -4)$

$-4 = a(1+1)^2 + 4 \Rightarrow a = -2$, so $y = -2(x+1)^2 + 4$.

c) vertex $(0, 1)$, point $(3, 10)$

$10 = 9a + 1 \Rightarrow a = 1$, so $y = x^2 + 1$.

QUESTION 4

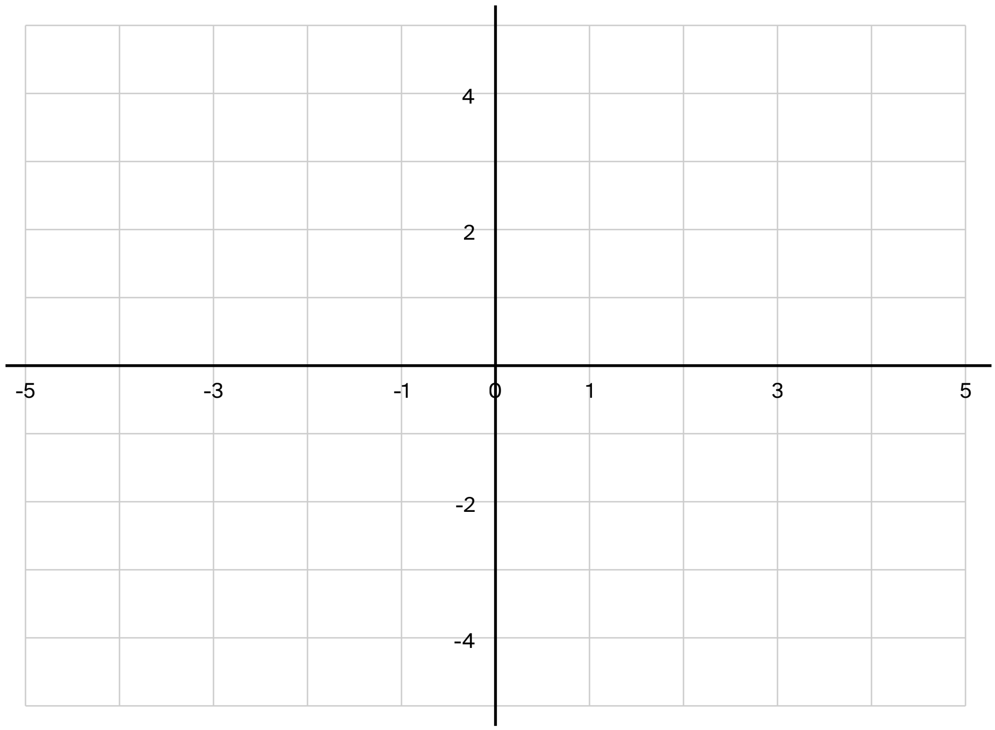
Harper says $y = (x+3)^2 - 2$ is $y = x^2$ moved right 3 and down 2. Check the claim by finding the vertex and substituting one easy *x*-value. Explain the horizontal shift clearly.

The vertex is $(-3, -2)$, so the graph moves left 3 and down 2. At $x = -3$, $y = (-3+3)^2 - 2 = -2$. The expression inside the square becomes zero at $x = -3$.

QUESTION 5

Sketch both functions on the same coordinate grid. Label each vertex and one symmetric pair of points.

a) $y = (x-1)^2 - 4$ **b)** $y = -2(x+2)^2 + 3$



a) vertex $(1, -4)$; for example, $(0, -3)$ and $(2, -3)$. **b)** vertex $(-2, 3)$; for example, $(-3, 1)$ and $(-1, 1)$.

QUESTION 6

Create a quadratic function with vertex $(3, -5)$ that opens down and is narrower than $y = x^2$. Give one point on your graph and explain how you know it works.

Connecting Quadratic Forms (Key)

Name Block Date

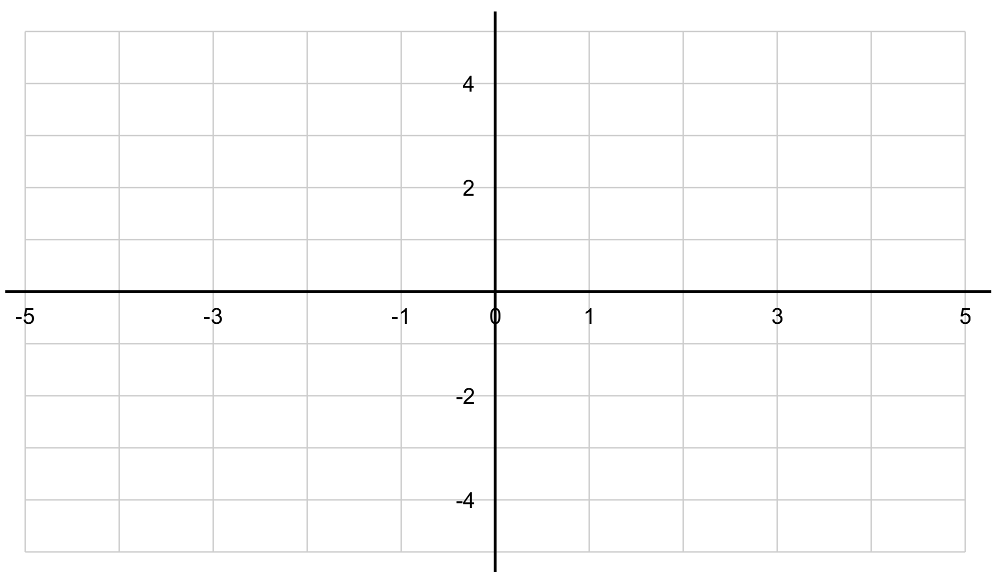
QUESTION 1

These three equations describe the same graph: $f(x) = x^2 - 2x - 3$, $f(x) = (x-3)(x+1)$, and $f(x) = (x-1)^2 - 4$. Circle the form you would use first to find the zeros, the vertex, and the *y*-intercept. Then list those features.

Zeros: factored form, $x = -1, 3$. Vertex: vertex form, $(1, -4)$. *y*-intercept: standard form, $(0, -3)$.

QUESTION 2

Sketch that graph. Label the vertex and all intercepts.



Plot $(-1, 0)$, $(3, 0)$, $(1, -4)$, and $(0, -3)$. The graph opens up and is symmetric about $x = 1$.

QUESTION 3

Rewrite each equation in the requested form.

a) $(x-2)(x+6)$ to standard form

b) $2(x-1)^2-8$ to standard and factored form

c) x^2+6x+5 to factored and vertex form

a) $(x-2)(x+6)=x^2+6x-2x-12=\ x^2+4x-12$.

b) $2(x-1)^2-8=2x^2-4x-6$. Factoring gives $2(x-3)(x+1)$.

c) $x^2+6x+5=\ (x+1)(x+5)$. Completing the square gives $(x+3)^2-4$.

QUESTION 4

Noah says $(x-4)(x+2)$ has vertex $(4,-2)$ because those are the numbers he sees. Find the actual vertex. What do the 4 and -2 tell you?

The zeros are 4 and -2, so the axis is halfway between them at $x=1$. Substitution gives $y=(1-4)(1+2)=-9$. The vertex is $(1,-9)$.

QUESTION 5

Choose the form you would use first to answer each question about $g(x) = 2x^2 - 12x + 10 = 2(x-1)(x-5) = 2(x-3)^2 - 8$. Explain each choice in a short sentence.

Where does the graph cross the *x*-axis?

What is its minimum value?

Where does it cross the *y*-axis?

Use factored form for the zeros $1, 5$; vertex form for the minimum -8 at $x = 3$; standard form for the *y*-intercept 10.

QUESTION 6

Make one quadratic in standard, factored, and vertex form. Use the forms to give its zeros, vertex, and *y*-intercept. Check that all three equations are equivalent.

Quadratic Function Word Problems (Key)

Name Block Date

QUESTION 1

A toy rocket follows $h(t) = -5(t-2)^2 + 20$, where *h* is height in metres and *t* is time in seconds. Find its greatest height, when it reaches that height, and when it is on the ground.

What do you know?

What are you finding?

Which form helps?

What will you check?

The vertex gives a greatest height of $20m$ at $t = 2s$. For the ground, $-5(t-2)^2 + 20 = 0$, so $(t-2)^2 = 4$ and $t = 0, 4$.

QUESTION 2

A community garden has 60 m of fencing for a rectangle. If one side is *x* metres, its area is $A(x) = x(30 - x)$. Find the dimensions that give the greatest area. Explain why your answer makes sense.

The zeros are 0 and 30, so the vertex is halfway at $x = 15$. Both sides are 15 m and the greatest area is $225m^2$.

QUESTION 3

A bridge arch is modelled by $y=-\frac{1}{4}(x-6)^2+9$, with distances in metres. The road is at $y=0$. Find the height and width of the opening.

The vertex gives height $9m$. Setting $y=0$ gives $(x-6)^2=36$, so $x=0,12$. The width is $12m$.

QUESTION 4

A small theatre models its ticket revenue by $R(p)=-50(p-18)^2+16200$, where *p* is the ticket price in dollars. What ticket price gives the greatest revenue? What is that revenue? Explain how the equation shows both answers.

The equation is in vertex form. Its vertex is $(18,16200)$, and the negative leading number means the graph opens down.

The greatest revenue is **$16,200** when tickets cost **$18**.

QUESTION 5

A soccer ball is kicked from a 1 m platform. Its height is $h(t) = -5t^2 + 20t + 1$. Find the greatest height. Then find the two times when the ball is 16 m high and say what those times mean.

The vertex occurs at $t = 2$, and $h(2) = 21m$. For 16 m: $-5t^2 + 20t + 1 = 16$, so $t^2 - 4t + 3 = 0$ and $t = 1, 3$. The ball passes 16 m once going up and once coming down.

Zeros, Factors, and Graphs (Key)

Name Block Date

QUESTION 1

Without solving anything new, connect these three statements: $(x-5)(x+2)=0$, the zeros are 5 and -2, and the graph crosses the *x*-axis at $(5,0)$ and $(-2,0)$. What stays the same?

They all describe where the function value is zero. A zero is an *x*-value, and the matching intercept is a point.

QUESTION 2

Solve each equation by factoring. Check each answer in the original equation.

a) $x^2-3x-10=0$	**b)** $2x^2+5x-3=0$
c) $3x^2-12x=0$	**d)** $x^2+4=0$

a) $(x-5)(x+2)=0$, so $x=5,-2$.

b) $(2x-1)(x+3)=0$, so $x=\frac{1}{2},-3$.

c) $3x(x-4)=0$, so $x=0,4$.

d) $x^2=-4$ has **no real solution**.

Substituting each real answer into its original equation gives zero.

QUESTION 3

The graph of $y = (x+4)(x-2)$ has two *x*-intercepts. Find them, the axis of symmetry, and the vertex.

The factors are zero at $x = -4$ and $x = 2$, so the intercepts are $(-4, 0)$ and $(2, 0)$.

The axis is halfway between the zeros: $x = \frac{-4+2}{2} = -1$.

$y = (-1+4)(-1-2) = -9$, so the vertex is $(-1, -9)$.

QUESTION 4

Mika solves $x(x-7) = 18$ by writing $x = 0$ or $x = 7$. What went wrong? Fix the solution.

The zero-product rule only works when the product equals zero.

$x(x-7) = 18$
$x^2 - 7x - 18 = 0$
$(x-9)(x+2) = 0$

So $x = 9$ or $x = -2$.

QUESTION 5

Find all values of *k* that make $x^2 + kx + 16 = 0$ factor into two equal factors. Explain how the graph would look in each case.

$(x+4)^2 = x^2 + 8x + 16$, so $k = 8$.

$(x-4)^2 = x^2 - 8x + 16$, so $k = -8$.

Each graph touches the *x*-axis once, at $x = -4$ or $x = 4$.

QUESTION 6

Write a quadratic equation with zeros -3 and 7 that also passes through (1,-24). Show how you chose the leading number.

$y = a(x+3)(x-7)$. Using $(1, -24)$ gives $-24 = a(4)(-6)$, so $a = 1$. One equation is $x^2 - 4x - 21 = 0$.

Square Roots and Completing the Square (Key)

Name Block Date

QUESTION 1

Compare $(x-3)^2 = 16$ and $x - 3 = 4$. How many answers does each equation have? Explain the difference.

The squared equation has $x - 3 = \pm 4$, so $x = 7, -1$. The second equation has only $x = 7$. Squaring hides whether the inside was positive or negative.

QUESTION 2

Solve by taking square roots.

a) $(x+1)^2 = 25$

b) $(x-3)^2 = 12$

c) $4x^2 = 25$

d) $2(x-4)^2 + 6 = 0$

a) $x + 1 = \pm 5$, so $x = 4, -6$.

b) $x - 3 = \pm\sqrt{12} = \pm 2\sqrt{3}$, so $x = 3 \pm 2\sqrt{3}$.

c) $x^2 = \frac{25}{4}$, so $x = \pm\frac{5}{2}$.

d) $(x-4)^2 = -3$, so there is **no real solution**.

QUESTION 3

Complete the square, then write each expression in vertex form.

a) $x^2 + 8x + 5$

b) $2x^2 - 12x + 7$

c) $-x^2 - 4x + 9$

---

a) $x^2 + 8x + 5 = (x^2 + 8x + 16) - 16 + 5 = (x + 4)^2 - 11.$

b) $2(x^2 - 6x) + 7 = 2[(x - 3)^2 - 9] + 7 = 2(x - 3)^2 - 11.$

c) $-(x^2 + 4x) + 9 = -[(x + 2)^2 - 4] + 9 = -(x + 2)^2 + 13.$

QUESTION 4

Solve $x^2 + 6x - 7 = 0$ by completing the square. Then check your answers by factoring.

$x^2 + 6x = 7$
$x^2 + 6x + 9 = 16$
$(x + 3)^2 = 16$

$x + 3 = \pm 4$, so $x = 1, -7.$

Check by factoring: $x^2 + 6x - 7 = (x - 1)(x + 7).$

QUESTION 5

Ari writes $x^2 - 10x + 7 = (x-5)^2 + 7$. Expand the right side and check it. Then fix the vertex form.

The right side expands to $x^2 - 10x + 32$. Since $x^2 - 10x + 7 = (x-5)^2 - 18$, the correct form is $(x-5)^2 - 18$.

QUESTION 6

Choose a value of *c* so $x^2 + 12x + c$ is a perfect square. Then choose a different value of *c* so the expression has no real zeros. Explain both choices.

Quadratic Formula and Choosing a Method (Key)

Name Block Date

QUESTION 1

For each equation, choose the method you would try first: factoring, square roots, completing the square, or the quadratic formula. Do not solve yet. Give one reason for each choice.

a) $x^2 - 9x + 20 = 0$

b) $3(x+2)^2 = 21$

c) $2x^2 + 3x - 4 = 0$

d) $x^2 + 6x + 1 = 0$

a) factoring. **b)** square roots. **c)** quadratic formula. **d)** completing the square or quadratic formula.

QUESTION 2

Use the quadratic formula. Give exact answers, then decimal answers to the nearest hundredth.

a) $2x^2 + 3x - 4 = 0$

b) $x^2 - 4x - 3 = 0$

c) $3x^2 + 2x + 5 = 0$

a) $a = 2, b = 3, c = -4$

$x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-4)}}{2(2)} = \frac{-3 \pm \sqrt{41}}{4}$

$x \approx 0.85, -2.35$

b) $x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(-3)}}{2} = \frac{4 \pm \sqrt{28}}{2}$

$x = 2 \pm \sqrt{7} \approx 4.65, -0.65$

c) $D = 2^2 - 4(3)(5) = -56 < 0$, so there are **no real solutions**.

QUESTION 3

Without solving, use the discriminant to say whether each equation has two, one, or no real solutions.

a) $x^2 + x - 12 = 0$

b) $4x^2 + 12x + 9 = 0$

c) $x^2 - 2x + 5 = 0$

a) $D = 1^2 - 4(1)(-12) = 49 > 0$, so there are **two real solutions**.

b) $D = 12^2 - 4(4)(9) = 0$, so there is **one real solution**.

c) $D = (-2)^2 - 4(1)(5) = -16 < 0$, so there are **no real solutions**.

QUESTION 4

Jules uses $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ on $-2x^2 + 5x + 3 = 0$ but sets $a = 2$. Follow that choice far enough to see the problem, then solve correctly.

Using $a = 2$ gives $D = 5^2 - 4(2)(3) = 1$ and $x = \frac{-5 \pm 1}{4} = -1, -\frac{3}{2}$. Neither value satisfies the original equation.

The leading coefficient is $a = -2$. Then $D = 5^2 - 4(-2)(3) = 49$.

$x = \frac{-5 \pm 7}{-4}$, so $x = -\frac{1}{2}, 3$.

QUESTION 5

Solve each equation using an efficient method. Name the method you chose.

a) $x^2 - 7x + 12 = 0$

b) $(x-3)^2 = 5$

c) $3x^2 + x - 5 = 0$

a) Factoring: $(x-3)(x-4) = 0$, so $x = 3, 4$.

b) Square roots: $x - 3 = \pm\sqrt{5}$, so $x = 3 \pm \sqrt{5}$.

c) Quadratic formula: $D = 1^2 - 4(3)(-5) = 61$, so $x = \frac{-1\pm\sqrt{61}}{6}$.

QUESTION 6

Compare $(x-2)(x+5) = 0$ and $x^2 + 3x = 10$. Show that they have the same solutions. Which equation is ready to solve first?

The first equation is already factored, so it gives $x = 2, -5$.

For the second equation, move 10 to the left:

$x^2 + 3x - 10 = 0$
$(x-2)(x+5) = 0$

It has the same solutions. The first equation is ready to solve first.

Quadratic Equation Word Problems (Key)

Name Block Date

QUESTION 1

A rectangular mural has area 96 m^2. Its length is 4 m more than its width. Find its dimensions.

What do you know?

What are you finding?

Choose a variable

Write a useful equation

Let the width be w. Then $w(w+4)=96$, so $w^2+4w-96=0$.

$(w+12)(w-8)=0$, so $w=-12$ or $w=8$. A width must be positive.

The mural is **8 m by 12 m**.

QUESTION 2

A ball is thrown upward from a balcony. Its height is $h(t)=-5t^2+15t+20$ metres. When does it hit the ground? Ignore any time before it was thrown.

$-5t^2+15t+20=0$

Divide by -5: $t^2-3t-4=0$.
$(t-4)(t+1)=0$, so $t=4$ or $t=-1$.

Ignore the time before the throw. The ball hits the ground after **4 s**.

QUESTION 3

Two consecutive positive odd integers have a product of 143. Find the integers.

Let the smaller odd integer be n. The next odd integer is $n+2$.

$n(n+2)=143$
$n^2+2n-143=0$
$(n-11)(n+13)=0$

The positive integers are **11 and 13**.

QUESTION 4

A photo is 18 cm by 24 cm. A border of equal width is placed inside all four edges, leaving 280 cm² of visible photo. Find the border width.

$(18-2x)(24-2x)=280$

$4x^2-84x+432=280$
$x^2-21x+38=0$
$(x-2)(x-19)=0$

$x=19$ does not fit inside the photo, so the border is **2 cm** wide.

QUESTION 5

A right triangle has legs that differ by 7 cm and a hypotenuse of 13 cm. Find the two leg lengths.

Let the shorter leg be x. Then $x^2 + (x+7)^2 = 169$.

$$2x^2 + 14x - 120 = 0$$
$$x^2 + 7x - 60 = 0$$
$$(x+12)(x-5) = 0$$

A length must be positive. The legs are **5 cm and 12 cm**.

QUESTION 6

A rectangular dog run uses a wall as one side and 34 m of fencing for the other three sides. What dimensions give an area of 120 m^2? There are two possible runs. Find both.

If each short side is x, the long side is $34 - 2x$. Solve $x(34-2x) = 120$: $x^2 - 17x + 60 = 0$, so $x = 5, 12$. The runs are 5 m by 24 m and 12 m by 10 m.

Restrictions and Equivalent Expressions (Key)

Name Block Date

QUESTION 1

Compare $\frac{x^2-9}{x-3}$ and $x+3$. Try $x=4$ and $x=3$ in both. In what way are they the same, and what is different?

At $x=4$, both equal 7. At $x=3$, the first is undefined while the second equals 6. They agree only when $x \neq 3$.

QUESTION 2

State every restriction before simplifying.

a) $\frac{x^2-16}{x-4}$

b) $\frac{x(x-2)}{x(x+5)}$

c) $\frac{(x+3)^2}{x^2-9}$

a) Restriction: $x \neq 4$.

$\frac{(x-4)(x+4)}{x-4} = x+4$

b) Restrictions: $x \neq 0, -5$.

$\frac{x(x-2)}{x(x+5)} = \frac{x-2}{x+5}$

c) Restrictions: $x \neq -3, 3$.

$\frac{(x+3)^2}{(x-3)(x+3)} = \frac{x+3}{x-3}$

QUESTION 3

Simplify. Keep the restrictions from the original expression.

a) $\frac{x^2-1}{x^2+x-2}$

b) $\frac{2x^2-12x+18}{x^2+x-12}$

c) $\frac{x^2-10x+25}{x^2-25}$

---

a) $\frac{(x-1)(x+1)}{(x-1)(x+2)} = \frac{x+1}{x+2}$, with $x \neq 1, -2.$

b) $\frac{2(x-3)^2}{(x+4)(x-3)} = \frac{2(x-3)}{x+4}$, with $x \neq 3, -4.$

c) $\frac{(x-5)^2}{(x-5)(x+5)} = \frac{x-5}{x+5}$, with $x \neq -5, 5.$

QUESTION 4

Sam crosses out the *x* in $\frac{x+6}{x}$ and writes 6. Use $x = 2$ to check the claim. Then explain when cancelling is allowed.

At $x = 2$, $\frac{x+6}{x} = \frac{8}{2} = 4$, not 6.

You can cancel a factor that multiplies the whole numerator and denominator. In $x + 6$, the x is one term in a sum, so **no cancellation is allowed**.

QUESTION 5

Find values of *a* and *b* so $\frac{x^2+ax+b}{x+2}$ simplifies to $x+5$, with one restriction. Show how you know.

$(x+2)(x+5)=x^2+7x+10$, so $a=7$ and $b=10$. The restriction is $x \neq -2$.

QUESTION 6

Compare $\frac{(x-1)(x+2)}{x-1}$ and $\frac{(x-3)(x+2)}{x-3}$. What do both simplify to? Why are the original expressions still different?

Both simplify to $x+2$.

The first expression has restriction $x \neq 1$. The second has restriction $x \neq 3$. The simplified rule matches, but each original expression is undefined at a different value.

Operations and Equations (Key)

Name Block Date

QUESTION 1

Which pair can be added without changing either denominator: $\frac{2}{x+1} + \frac{5}{x+1}$ or $\frac{2}{x+1} + \frac{5}{x-1}$? Explain what the denominator is doing.

The first pair already has a common denominator and becomes $\frac{7}{x+1}$. The denominator names equal-sized algebraic parts; unlike denominators must be rewritten before their numerators can be combined.

QUESTION 2

Simplify each result and state restrictions.

a) $\frac{3}{x+2} + \frac{4}{x+2}$

b) $\frac{2}{x-1} + \frac{3}{x+1}$

c) $\frac{x^2-1}{x^2-2x} \cdot \frac{x}{x-1}$

d) $\frac{\frac{3x}{2x-2}}{\frac{x}{x+2}}$

a) $\frac{3}{x+2} + \frac{4}{x+2} = \frac{7}{x+2}, x \neq -2.$

b) $\frac{2(x+1)+3(x-1)}{(x-1)(x+1)} = \frac{5x-1}{(x-1)(x+1)}, x \neq -1, 1.$

c) $\frac{(x-1)(x+1)}{x(x-2)} \cdot \frac{x}{x-1} = \frac{x+1}{x-2}, x \neq 0, 1, 2.$

d) $\frac{3x}{2(x-1)} \cdot \frac{x+2}{x} = \frac{3(x+2)}{2(x-1)}, x \neq -2, 0, 1.$

QUESTION 3

Solve. Check each answer against the original restrictions.

a) $\frac{3}{x}+\frac{1}{4}=\frac{5}{8}$

b) $\frac{2}{x-1}=\frac{4}{x+1}$

c) $\frac{1}{x-2}+\frac{1}{x+2}=\frac{10}{21}$

a) $x\neq 0$. Multiply by $8x$: $24+2x=5x$, so $x=8$.

b) $x\neq 1,-1$. $2(x+1)=4(x-1)$, so $x=3$.

c) $x\neq 2,-2$. Multiply by $21(x-2)(x+2)$:
$21(x+2)+21(x-2)=10(x^2-4)$
$5x^2-21x-20=0$
$(5x+4)(x-5)=0$
$x=5,-\frac{4}{5}$. Both are allowed.

QUESTION 4

Priya solves $\frac{x+2}{x-3}=1$ by cancelling the *x* terms and getting $\frac{2}{-3}=1$. Check the original equation with a few values, then solve it correctly.

Cross-multiplying gives $x+2=x-3$, which becomes $2=-3$. That contradiction means there is **no solution**. Terms cannot be cancelled across addition.

QUESTION 5

Solve $\frac{x}{x-2} = \frac{6}{x^2-4} + 1$. State restrictions first and check the result.

Restrictions: $x \neq \pm 2$. Multiplying by $(x-2)(x+2)$ gives $x(x+2) = 6 + (x^2 - 4)$, so $2x = 2$ and $x = 1$, which checks.

QUESTION 6

Find a value of *k* so $\frac{3}{x-1} + \frac{k}{x+1} = \frac{5x+1}{x^2-1}$ is true for every allowed *x*. Explain how you found it.

$3(x+1) + k(x-1) = (3+k)x + (3-k)$. Matching $5x + 1$ gives $k = 2$.

Rational Expression Word Problems (Key)

Name Block Date

QUESTION 1

One hose fills a tank in 6 hours. A second hose fills it in 4 hours. How long do they take together?

What is each rate?

What rate is needed?

Write an equation

What will the answer mean?

Their combined rate is $\frac{1}{6}+\frac{1}{4}=\frac{5}{12}$ tank per hour. The time is $\frac{1}{\frac{5}{12}}=\frac{12}{5}=2.4$ hours, or 2 h 24 min.

QUESTION 2

A paddler travels 12 km downstream and 12 km upstream. The river current is 2 km/h. The whole trip takes 5 hours. Find the paddler’s speed in still water.

Let the still-water speed be $v>2$ km/h.

$\frac{12}{v+2}+\frac{12}{v-2}=5$
$12(v-2)+12(v+2)=5(v+2)(v-2)$
$5v^2-24v-20=0$

$v=\frac{24\pm\sqrt{976}}{10}=\frac{12\pm2\sqrt{61}}{5}$. The negative value is not a speed.

The paddler's speed is $\frac{12+2\sqrt{61}}{5}\approx 5.52$ **km/h**.

QUESTION 3

A printer can finish a job in 30 minutes. Working with a second printer, the job takes 18 minutes. How long would the second printer take alone?

$\frac{1}{30} + \frac{1}{t} = \frac{1}{18}$

$\frac{1}{t} = \frac{1}{18} - \frac{1}{30} = \frac{1}{45}$

The second printer takes **45 minutes**.

QUESTION 4

A cyclist rides 24 km to a park, then returns along the same route at a speed 4 km/h faster. The return trip takes 30 minutes less. Find both speeds.

Let the first speed be $v > 0$ km/h. The return speed is $v + 4$ km/h.

$\frac{24}{v} - \frac{24}{v+4} = \frac{1}{2}$

$48(v + 4) - 48v = v(v + 4)$

$v^2 + 4v - 192 = 0$

$(v - 12)(v + 16) = 0$

Reject the negative value. The speeds are **12 km/h and 16 km/h**.

QUESTION 5

A class buys a $360 piece of equipment. If 6 more students join, each person pays $5 less. How many students were in the original group?

Let the original number be n. Then $n > 0$.

$\frac{360}{n} - \frac{360}{n+6} = 5$
$360(n+6) - 360n = 5n(n+6)$
$n^2 + 6n - 432 = 0$
$(n-18)(n+24) = 0$

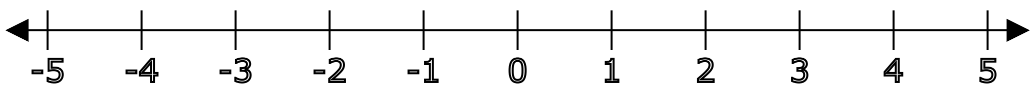
Reject $n = -24$. The original group had **18 students**.

Linear Inequalities and Interval Notation (Key)

Name Block Date

QUESTION 1

Which values from $\{-3, -1, 0, 2, 5\}$ make $2x - 1 < 5$ true? Put them on the number line. What boundary separates the values that work from the values that do not?



$-3, -1, 0, 2$ work. The boundary is $x = 3$, which is not included.

QUESTION 2

Solve each inequality. Give the answer in inequality notation and interval notation.

a) $3x + 4 < 19$

b) $-2x + 5 \geq -3$

c) $-7 < 2x - 1 \leq 3$

d) $x \leq -2$ or $x > 3$

a) $3x < 15$, so $x < 5$ and $(-\infty, 5)$.

b) $-2x \geq -8$. Divide by -2 and reverse the sign: $x \leq 4$, or $(-\infty, 4]$.

c) Add 1, then divide by 2: $-6 < 2x \leq 4$, so $-3 < x \leq 2$, or $(-3, 2]$.

d) $x \leq -2$ **or** $x > 3$, so $(-\infty, -2] \cup (3, \infty)$.

QUESTION 3

Write each number-line description in interval notation.

a) from -1, included, to 4, not included

b) less than 2 or greater than 6

c) at least 3

a) $[-1, 4)$. **b)** $(-\infty, 2) \cup (6, \infty)$. **c)** $[3, \infty)$.

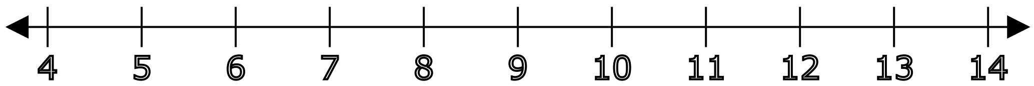
QUESTION 4

Eli solves $-4x > 12$ and writes $x > -3$. Test one value from Eli's answer, then fix it. Why does the sign change direction?

Testing $x = 0$ gives $0 > 12$, false. The answer is $x < -3$. Multiplying or dividing both sides by a negative reverses their order.

QUESTION 5

Solve $2 < \frac{x+1}{3} \leq 5$. Show the answer on a number line and in interval notation.



$6 < x + 1 \leq 15$, so $5 < x \leq 14$. Interval: $(5, 14]$.

QUESTION 6

Compare $-2 \leq x < 5$ and $0 \leq x + 2 < 7$. Do they have the same solution? Show how you know and give the interval.

Subtract 2 from every part of the second inequality:

$0 - 2 \leq x + 2 - 2 < 7 - 2$
$-2 \leq x < 5$

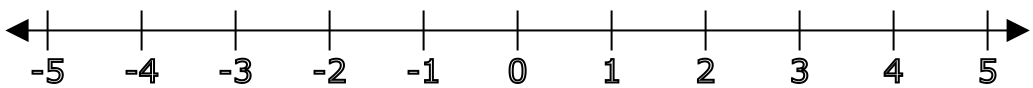
They have the same solution: $[-2, 5)$.

Quadratic Inequalities and Sign Analysis (Key)

Name Block Date

QUESTION 1

The factors in $(x-2)(x+3)$ change sign at -3 and 2. Pick one number from each interval and decide whether the product is positive or negative.



For $x<-3$, both factors are negative, so the product is positive. For $-3<x<2$, the signs differ, so it is negative. For $x>2$, both are positive.

QUESTION 2

Solve each inequality. Show the boundary values and give interval notation.

a) $(x-2)(x+3)>0$

b) $x^2-3x-4\le 0$

c) $2x^2-2x-12\ge 0$

a) Boundaries: $x=-3,2$. The product is positive outside them, so $(-\infty,-3)\cup(2,\infty)$.

b) $x^2-3x-4=(x-4)(x+1)$. The product is non-positive between the boundaries, so $[-1,4]$.

c) $2x^2-2x-12=2(x-3)(x+2)$. The product is non-negative outside the boundaries, so $(-\infty,-2]\cup[3,\infty)$.

QUESTION 3

Solve. Factor first when it helps.

a) $x^2 - 3x - 10 < 0$

b) $2x^2 - 5x - 3 \geq 0$

c) $x^2 + 4x + 7 > 0$

a) $x^2 - 3x - 10 = (x-5)(x+2)$. The product is negative between the zeros, so $(-2, 5)$.

b) $2x^2 - 5x - 3 = (2x+1)(x-3)$. The product is non-negative outside the zeros, so $(-\infty, -\frac{1}{2}] \cup [3, \infty)$.

c) $x^2 + 4x + 7 = (x+2)^2 + 3$, which is always positive. The answer is **all real numbers**.

QUESTION 4

Zoey solves $x^2 < 9$ by taking square roots and writing $x < 3$. Find a value that exposes the problem, then give the full answer.

$x = -10$ satisfies $x < 3$ but not $x^2 < 9$. The full answer is $-3 < x < 3$.

QUESTION 5

Find all values of *k* for which $x^2 - 6x + k > 0$ is true for every real *x*. Explain using either the vertex or the discriminant.

The minimum occurs at $x = 3$ and equals $k - 9$. It must be positive, so $k > 9$. Equivalently, the discriminant must be negative.

QUESTION 6

Draw a parabola whose solution to $f(x) \leq 0$ is $[-4, 1]$. Write one possible equation for it.

Inequality Word Problems (Key)

Name Block Date

QUESTION 1

A student can spend at most \$42 on a ride-share. The trip costs \$6 plus \$2.40 per kilometre. How far can the student travel?

What is fixed?

What changes?

What does "at most" mean?

Write an inequality

Let distance be $d \geq 0$. Solve $6 + 2.40d \leq 42$, giving $d \leq 15$. The student can travel at most 15 km.

QUESTION 2

A rectangular pen uses 40 m of fencing. One side is x metres, so its area is $A = x(20 - x)$. For which side lengths is the area at least 75 m²?

$x(20 - x) \geq 75$ becomes $x^2 - 20x + 75 \leq 0$.

$(x - 5)(x - 15) \leq 0$. The product is non-positive between the boundary values.

So $5 \leq x \leq 15$ metres.

QUESTION 3

A small business models daily profit by $P(n) = -2(n-10)(n-40)$, where n items are sold. For what sales numbers is the profit positive? Remember that sales must be whole numbers.

The expression is positive between its zeros: $10 < n < 40$. For whole-number sales, $n = 11, 12, \ldots, 39$.

QUESTION 4

The stopping distance of a bicycle is modelled by $d = 0.05v^2 + 0.3v$, where v is speed in km/h and d is metres. A path has 14 m of clear space. Find the greatest safe speed to the nearest km/h.

Solve $0.05v^2 + 0.3v \leq 14$. Multiplying by 20 gives $v^2 + 6v - 280 \leq 0$, with boundaries $v = 14, -20$. Since speed is non-negative, $0 \leq v \leq 14$. Greatest safe speed: 14 km/h.

QUESTION 5

A concert hall earns $R(p) = -100(p-25)^2 + 62500$ dollars when tickets cost p dollars. The event needs at least $60,000 in revenue. Find the allowed ticket-price range.

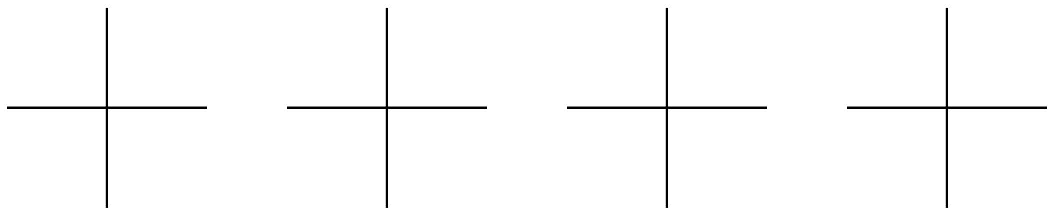
$-100(p-25)^2 + 62500 \geq 60000$ gives $(p-25)^2 \leq 25$. Thus $20 \leq p \leq 30$ dollars.

Angles in Standard Position (Key)

Name Block Date

QUESTION 1

Start on the positive *x*-axis. For each turn, say which quadrant or axis the terminal arm reaches. Sketch each turn on its axes.



80° 170° 270° -45°

80°**: quadrant I**. 170°**: quadrant II**. 270°**: negative *y*-axis**. -45°**: quadrant IV**.

QUESTION 2

Find one positive and one negative coterminal angle for each angle.

a) 30°

b) -225°

c) 600°

a) $30^\circ + 360^\circ = 390^\circ$ and $30^\circ - 360^\circ = -330^\circ$.

b) $-225^\circ + 360^\circ = 135^\circ$ and $-225^\circ - 360^\circ = -585^\circ$.

c) $600^\circ - 360^\circ = 240^\circ$ and $600^\circ - 720^\circ = -120^\circ$.

QUESTION 3

Find the reference angle and quadrant.

a) 145°

b) 220°

c) -30°

d) 555°

a) QII, reference angle $180^\circ - 145^\circ = 35^\circ$.

b) QIII, reference angle $220^\circ - 180^\circ = 40^\circ$.

c) QIV, reference angle 30°.

d) $555^\circ - 360^\circ = 195^\circ$, so QIII with reference angle $195^\circ - 180^\circ = 15^\circ$.

QUESTION 4

Kai says the reference angle for -140° is 140° because reference angles are positive. Sketch the angle and fix the answer.

A coterminal angle is $-140^\circ + 360^\circ = 220^\circ$, which is in quadrant III.

The reference angle is $220^\circ - 180^\circ = 40^\circ$.

QUESTION 5

An angle in standard position has a reference angle of 28°. Find every possible angle from 0° to 360°.

Use the reference angle in all four quadrants:

28°, $180^\circ - 28^\circ = 152^\circ$, $180^\circ + 28^\circ = 208^\circ$, and $360^\circ - 28^\circ = 332^\circ$.

The angles are $28^\circ, 152^\circ, 208^\circ, 332^\circ$.

QUESTION 6

Compare 40° and 400°. Draw both turns. Are they coterminal? Explain without relying only on your drawing.

$400^\circ - 40^\circ = 360^\circ$, which is one full turn.

The angles are **coterminal** because they differ by a multiple of 360°.

Radian Measure (Key)

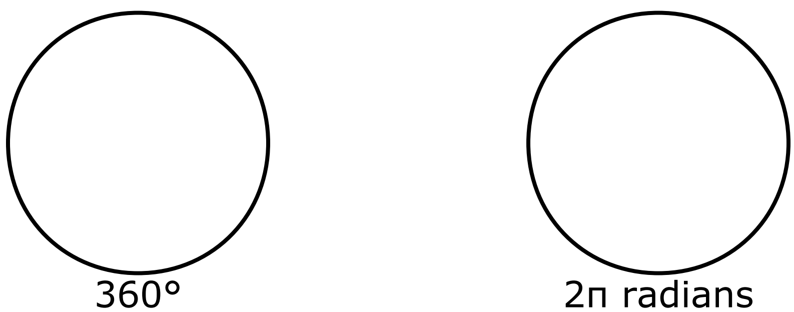
Name Block Date

QUESTION 1

Why do you think people settled on 360 degrees for one full turn? What makes 360 convenient? Write down two useful features and one drawback.

QUESTION 2

The circles show the same full turn in two systems. Degrees use 360. Radians use 2π. What do you notice before doing any conversions?



A full turn is the same rotation in both pictures. The labels give $360^\circ = 2\pi$ radians, so half and quarter turns can be found by dividing both labels by the same amount.

QUESTION 3

A radian compares arc length with radius. Imagine wrapping pieces, each one radius long, around a circle. A circle has circumference $2\pi r$. How many of those pieces fit? Use your answer to explain why a full turn is 2π radians.

The number of radius lengths is $\frac{2\pi r}{r} = 2\pi$. One radius-length arc gives 1 radian, so the full circumference gives 2π radians.

QUESTION 4

Build the common conversions by starting with a full turn and repeatedly taking halves.

0° = ____

360° = ____

180° = ____

90° = ____

45° = ____

$0^\circ = 0$, $360^\circ = 2\pi$, $180^\circ = \pi$, $90^\circ = \frac{\pi}{2}$, and $45^\circ = \frac{\pi}{4}$.

QUESTION 5

Convert each angle. Keep radian answers exact.

a) 30°

b) 120°

c) 225°

d) $\frac{5\pi}{3}$

e) $\frac{7\pi}{4}$

a) $30^\circ \cdot \frac{\pi}{180^\circ} = \frac{\pi}{6}$.

b) $120^\circ \cdot \frac{\pi}{180^\circ} = \frac{2\pi}{3}$.

c) $225^\circ \cdot \frac{\pi}{180^\circ} = \frac{5\pi}{4}$.

d) $\frac{5\pi}{3} \cdot \frac{180^\circ}{\pi} = 300^\circ$.

e) $\frac{7\pi}{4} \cdot \frac{180^\circ}{\pi} = 315^\circ$.

QUESTION 6

A student says $60^\circ = \frac{\pi}{60}$. Use the full-turn relationship to show why that cannot be right, then fix it.

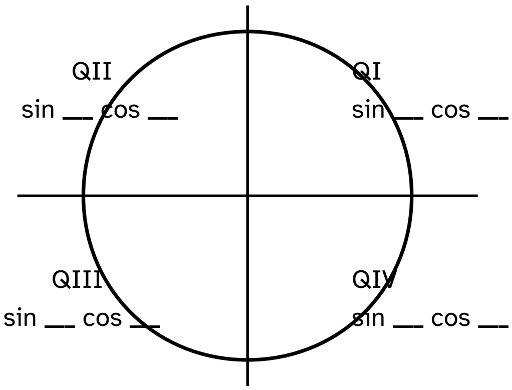
60° is one sixth of 360°, so it must be one sixth of 2π: $60^\circ = \frac{\pi}{3}$.

Special Angles and the Unit Circle (Key)

Name Block Date

QUESTION 1

On a unit circle, a point has coordinates $(cos\,\theta, sin\,\theta)$. Use the signs of *x* and *y* to fill in the sign of sine and cosine in each quadrant.



QI: sine +, cosine +. QII: sine +, cosine -. QIII: sine -, cosine -. QIV: sine -, cosine +.

QUESTION 2

Find the exact sine, cosine, and tangent.

a) 30°

b) 135°

c) $\frac{4\pi}{3}$

d) $\frac{3\pi}{2}$

a) QI with reference angle 30°: $sin = \frac{1}{2}, cos = \frac{\sqrt{3}}{2}, tan = \frac{\sqrt{3}}{3}$.

b) QII with reference angle 45°: $sin = \frac{\sqrt{2}}{2}, cos = -\frac{\sqrt{2}}{2}, tan = -1$.

c) QIII with reference angle $\frac{\pi}{3}$: $sin = -\frac{\sqrt{3}}{2}, cos = -\frac{1}{2}, tan = \sqrt{3}$.

d) On the negative *y*-axis: $sin = -1, cos = 0$**; tangent is undefined**.

QUESTION 3

Find every angle on $0^\circ \leq \theta < 360^\circ$ that matches the given condition.

a) $sin\,\theta = \frac{\sqrt{3}}{2}$

b) $tan\,\theta = -1$

c) $sin\,\theta = 0$

a) The reference angle is 60°. Sine is positive in QI and QII, so $\theta = 60^\circ, 120^\circ$.

b) The reference angle is 45°. Tangent is negative in QII and QIV, so $\theta = 135^\circ, 315^\circ$.

c) Sine is zero on the *x*-axis, so $\theta = 0^\circ, 180^\circ$.

QUESTION 4

Morgan says $cos\,150^\circ = \frac{\sqrt{3}}{2}$ because the reference angle is 30°. What part is right? What sign is missing?

The reference angle and magnitude are right. Since 150° is in quadrant II, cosine is negative: $cos\,150^\circ = -\frac{\sqrt{3}}{2}$.

QUESTION 5

Given $sin\,\theta = -\frac{5}{13}$ and θ is in quadrant IV, find exact values of $cos\,\theta$ and $tan\,\theta$.

A 5-12-13 triangle gives $cos\,\theta = \frac{12}{13}$ and $tan\,\theta = -\frac{5}{12}$.

QUESTION 6

Choose a special angle. Give four coterminal angles, two in degrees and two in radians, and show that all have the same sine and cosine.

Simple Trigonometric Equations (Key)

Name | Block | Date

QUESTION 1

For $sin\,\theta = \frac{1}{2}$, first find the reference angle. Then use the sign of sine to choose the quadrants. Why are there two answers from 0° to 360°?

The reference angle is 30°. Sine is positive in quadrants I and II, so $\theta = 30^\circ, 150^\circ$.

QUESTION 2

Solve from $0^\circ \leq \theta < 360^\circ$.

a) $cos\,\theta = \frac{1}{2}$

b) $cos\,\theta = -\frac{1}{2}$

c) $tan\,\theta = -1$

d) $cos\,\theta = 0$

a) Reference angle 60°; cosine is positive in QI and QIV: $\theta = 60^\circ, 300^\circ$.

b) Reference angle 60°; cosine is negative in QII and QIII: $\theta = 120^\circ, 240^\circ$.

c) Reference angle 45°; tangent is negative in QII and QIV: $\theta = 135^\circ, 315^\circ$.

d) Cosine is zero on the *y*-axis: $\theta = 90^\circ, 270^\circ$.

QUESTION 3

Solve from $0 \leq \theta < 2\pi$.

a) $sin\,\theta = \frac{\sqrt{2}}{2}$

b) $tan\,\theta = \sqrt{3}$

c) $sin\,\theta = 0$

a) Reference angle $\frac{\pi}{4}$; sine is positive in QI and QII: $\theta = \frac{\pi}{4}, \frac{3\pi}{4}$.

b) Reference angle $\frac{\pi}{3}$; tangent is positive in QI and QIII: $\theta = \frac{\pi}{3}, \frac{4\pi}{3}$.

c) Sine is zero on the *x*-axis. On the half-open interval, $\theta = 0, \pi$.

QUESTION 4

Sasha solves $2\,cos\,\theta - 1 = 0$ and gives only 60°. Check that answer, then find what is missing on $0^\circ \leq \theta < 360^\circ$.

60° works, but cosine is also positive in quadrant IV. The other answer is 300°.

QUESTION 5

Solve $3\,sin\,\theta + 1 = 0$ for $0^\circ \leq \theta < 360^\circ$. Give answers to the nearest tenth of a degree.

$sin\,\theta = -\frac{1}{3}$. The reference angle is about 19.5°. Sine is negative in III and IV, so $\theta \approx 199.5^\circ, 340.5^\circ$.

QUESTION 6

Write a simple trig equation with exactly two solutions on $0^\circ \leq \theta < 360^\circ$. Give the solutions and explain how you know there are no others.

Angles and Circular Motion Word Problems (Key)

Name Block Date

QUESTION 1

A Ferris wheel has radius 12 m. A rider travels through an angle of $\frac{5\pi}{6}$ radians. How far does the rider move along the circle?

What do you know?

What are you finding?

Which units fit the formula?

Write the relationship

Use $s = r\theta$.

$s = 12 \cdot \frac{5\pi}{6} = 10\pi \approx 31.4$

The rider moves 10π **m, or about 31.4 m**.

QUESTION 2

A bicycle wheel has radius 0.35 m. How many complete turns does it make over 1.1 km? Give a sensible whole-number estimate.

The circumference is $2\pi(0.35) = 0.7\pi$ m. The distance is 1100 m.

$\frac{1100}{0.7\pi} \approx 500.2$

The wheel makes about **500 complete turns**.

QUESTION 3

A sprinkler rotates from 25° to 230°. Its water reaches 8 m. Find the area watered during the turn.

The turn is $230^\circ - 25^\circ = 205^\circ = \frac{41\pi}{36}$.

$A = \frac{1}{2}r^2\theta = \frac{1}{2}(8^2)\frac{41\pi}{36} = \frac{328\pi}{9} \approx 114.5$

The area is $\frac{328\pi}{9}$ **m^2, or about 114.5 m^2**.

QUESTION 4

The tip of a 9 cm clock hand moves 12 cm along its circular path. Find the angle turned in radians and degrees.

$\theta = \frac{s}{r} = \frac{12}{9} = \frac{4}{3}$ rad. In degrees, $\frac{4}{3} \cdot \frac{180}{\pi} \approx 76.4^\circ$.

QUESTION 5

A wheel turns at 90 revolutions per minute. Find its angular speed in radians per second. Then find the linear speed of a point 0.20 m from the centre.

$90 \cdot \frac{2\pi}{60} = 3\pi$ rad/s, so the angular speed is 3π **rad/s**.

$v = r\omega = 0.20(3\pi) = 0.6\pi \approx 1.88$

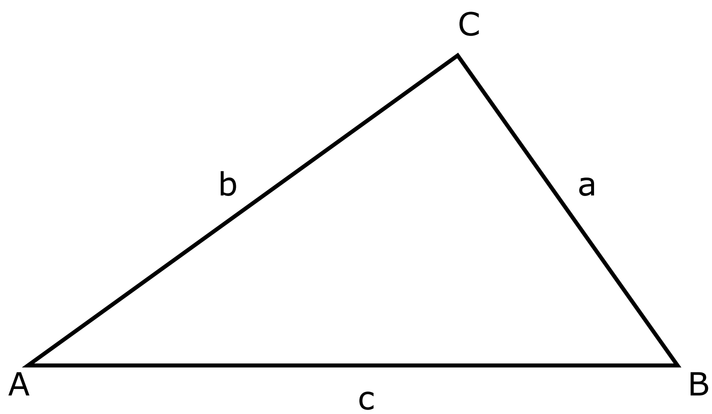
The linear speed is 0.6π **m/s, or about 1.88 m/s**.

Sine Law (Key)

Name Block Date

QUESTION 1

In the triangle, each side is opposite the angle with the same capital letter. Write the three opposite side-angle pairs. Why does keeping those pairs straight matter?



The pairs are $a \leftrightarrow A$, $b \leftrightarrow B$, and $c \leftrightarrow C$. The sine law compares each side with the sine of its opposite angle.

QUESTION 2

Find the missing value. Give lengths to the nearest tenth and angles to the nearest degree.

a) $A = 42^{\circ}, a = 12cm, B = 68^{\circ}$

b) $A = 35^{\circ}, a = 7m, b = 9m$

c) $B = 81^{\circ}, b = 15km, a = 11km$

a) $\frac{b}{sin\,68^{\circ}} = \frac{12}{sin\,42^{\circ}}$
$b = \frac{12\,sin\,68^{\circ}}{sin\,42^{\circ}} \approx 16.6$ **cm.**

b) $sin\,B = \frac{9\,sin\,35^{\circ}}{7} \approx 0.7375$. This gives $B_1 \approx 48^{\circ}$ or $B_2 = 180^{\circ} - B_1 \approx 132^{\circ}$. Both can fit with $A = 35^{\circ}$, so $B \approx 48^{\circ}$ **or** 132°.

c) $sin\,A = \frac{11\,sin\,81^{\circ}}{15}$, so $A \approx 46^{\circ}$. The second sine angle would make the total exceed 180°.

QUESTION 3

For $A = 35^\circ, a = 7, b = 9$, explain why a calculator gives one angle for *B* but the triangle may have two. Find both possible angles and decide whether each makes a triangle.

$sin\, B = \frac{9\, sin\, 35^\circ}{7} \approx 0.7375$

$B_1 \approx 47.5^\circ$ and $B_2 = 180^\circ - 47.5^\circ \approx 132.5^\circ$.

Both work because $35^\circ + B < 180^\circ$ in each case. There are **two possible triangles**.

QUESTION 4

Solve each triangle. State if there are zero, one, or two possible triangles.

a) $A = 60^\circ, a = 12, b = 9$

b) $A = 25^\circ, a = 5, b = 13$

c) $A = 40^\circ, a = 7, b = 8$

a) $sin\, B = \frac{9\, sin\, 60^\circ}{12}$, so $B \approx 41^\circ$. The other sine angle does not fit. Then $C \approx 79^\circ$ and $c \approx 13.6$.
One triangle.

b) $sin\, B = \frac{13\, sin\, 25^\circ}{5} \approx 1.10$. A sine cannot exceed 1, so **no triangle**.

c) $sin\, B = \frac{8\, sin\, 40^\circ}{7}$. This gives $B \approx 47^\circ$ or 133°.
$B \approx 47^\circ, C \approx 93^\circ, c \approx 10.9$, or $B \approx 133^\circ, C \approx 7^\circ, c \approx 1.4$. Two triangles.

QUESTION 5

Lee writes $\frac{a}{\sin B} = \frac{b}{\sin A}$. Use the labelled triangle to explain what does not match, then correct the equation.

Each side must stay with its opposite angle. The corrected relationship is $\frac{a}{\sin A} = \frac{b}{\sin B}$.

QUESTION 6

Create an SSA set of measurements that gives two possible triangles. Show both angle possibilities and explain why both fit.

Cosine Law and Choosing a Method (Key)

Name Block Date

QUESTION 1

For each set of information, choose sine law, cosine law, or ordinary right-triangle trig. Give one reason. Do not solve.

a) two sides and their included angle

b) two angles and one side

c) three sides

d) a right triangle with one acute angle and the hypotenuse

a) cosine law, SAS. **b)** sine law, an opposite pair is known. **c)** cosine law, SSS. **d)** right-triangle trig.

QUESTION 2

Use the cosine law. Give lengths to the nearest tenth and angles to the nearest degree.

a) $a = 8, b = 13, C = 50^\circ$

b) $a = 7, b = 9, c = 11.6$

c) $a = 12, c = 18, B = 62^\circ$

a) $c^2 = 8^2 + 13^2 - 2(8)(13)\,cos\,50^\circ$

$c \approx 10.0$.

b) $cos\,C = \frac{7^2+9^2-11.6^2}{2(7)(9)}$

$C \approx 92^\circ$.

c) $b^2 = 12^2 + 18^2 - 2(12)(18)\,cos\,62^\circ$

$b \approx 16.3$.

QUESTION 3

Solve the triangle with sides $a = 7, b = 10, c = 13$. Use a second relationship to check your angles.

$cos\,A = \frac{10^2+13^2-7^2}{2(10)(13)}$, so $A \approx 32.2^\circ$.

$cos\,B = \frac{7^2+13^2-10^2}{2(7)(13)}$, so $B \approx 49.6^\circ$.

$C = 180^\circ - A - B \approx 98.2^\circ$.

The angles are $A \approx 32.2^\circ, B \approx 49.6^\circ, C \approx 98.2^\circ$, and they total 180°.

QUESTION 4

Taylor uses $c^2 = a^2 + b^2$ for a triangle with $a = 6, b = 9, C = 70^\circ$. What assumption is hidden in that equation? Find c correctly.

The Pythagorean theorem assumes $C = 90^\circ$. Cosine law gives $c^2 = 6^2 + 9^2 - 2(6)(9)\,cos\,70^\circ$, so $c \approx 8.9$.

QUESTION 5

A triangle has sides 5 cm, 7 cm, and 11 cm. Find its largest angle. Explain how you knew which angle to find.

The largest angle is opposite the longest side, 11 cm. $cos\, C = \frac{5^2+7^2-11^2}{2(5)(7)} = -\frac{47}{70}$, so $C \approx 132^\circ$.

QUESTION 6

Two triangles each have sides of 8 units and 12 units around the included angle. One included angle is 40°; the other is 120°. Find the third side in each triangle. Which angle gives the longer third side?

For 40°: $c^2 = 8^2 + 12^2 - 2(8)(12)\, cos\, 40^\circ$, so $c \approx 7.8$.

For 120°: $c^2 = 8^2 + 12^2 - 2(8)(12)\, cos\, 120^\circ$, so $c \approx 17.4$.

The 120° **angle** gives the longer third side.

Non-right-triangle Word Problems (Key)

Name Block Date

QUESTION 1

Two trails leave a campsite at an angle of 68°. One hiker walks 4.2 km on one trail and another walks 6.1 km on the other. How far apart are they?

What do you know?

What are you finding?

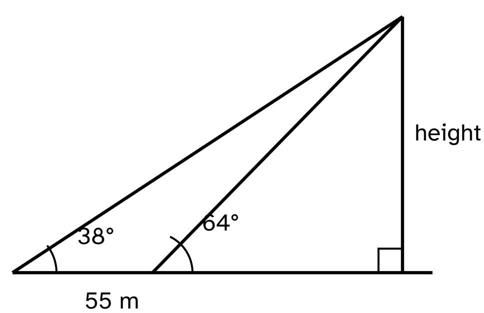
Sketch and label

Which law fits?

The known information is SAS, so use cosine law: $d^2 = 4.2^2 + 6.1^2 - 2(4.2)(6.1)\,cos\,68^{\circ}$. Thus $d \approx 6.0 km$.

QUESTION 2

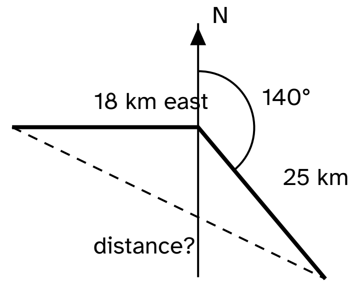
From a point on level ground, the angle of elevation to the top of a tower is 38°. From a second point 55 m closer to the tower, the angle is 64°. Find the tower height to the nearest metre.



The inside angle at the closer point is 116°, so the top angle is 26°. Sine law gives the slanted distance from the closer point: $\frac{d}{sin\,38^{\circ}} = \frac{55}{sin\,26^{\circ}}$, so $d \approx 77.2$. Height $= d\,sin\,64^{\circ} \approx 69m$.

QUESTION 3

A boat travels 18 km east, then travels 25 km on a bearing of 140°.
How far is it from its starting point?



East has bearing 90°, so the angle between the travel directions is 50°. The triangle's inside angle at the turn is 130°. Thus $d^2 = 18^2 + 25^2 - 2(18)(25)\,cos\,130^\circ$, so $d \approx 39.1 km$.

QUESTION 4

A surveyor sees two landmarks from point P. One is 320 m away and the other is 470 m away.
The angle between the sight lines is 52°. Find the distance between the landmarks.

$d^2 = 320^2 + 470^2 - 2(320)(470)\,cos\,52^\circ$, so $d \approx 372m$.

QUESTION 5

Two rescue stations A and B are 24 km apart. A distress signal is seen at 58° from the line AB at A and 47° from the line BA at B. Find the distance from each station to the signal.

The third angle is 75°. Sine law gives distance from A $= \frac{24\,sin\,47^\circ}{sin\,75^\circ} \approx 18.2km$ and from B $= \frac{24\,sin\,58^\circ}{sin\,75^\circ} \approx 21.1km.$

Compound Interest and Financial Growth (Key)

Name Block Date

QUESTION 1

A $1,000 investment earns 5% once per year. Predict whether the interest earned in year 2 will be less than, equal to, or greater than the interest earned in year 1. Then calculate both amounts.

Year 1 interest is $50, leaving $1,050. Year 2 interest is $52.50, which is greater because interest is earned on the earlier interest too.

QUESTION 2

Find the future value. Round money to the nearest cent.

a) $2,000 at 4% compounded yearly for 5 years

b) $1,500 at 6% compounded monthly for 3 years

c) $8,000 at 4.5% compounded quarterly for 6 years

a) $2000(1.04)^5 = \$2433.31$. **b)** $1500\left(1+\frac{0.06}{12}\right)^{36} = \$1795.02$. **c)** $8000\left(1+\frac{0.045}{4}\right)^{24} = \$10463.93$.

QUESTION 3

Find the original amount needed.

a) to have $5,000 after 8 years at 3% yearly

b) to have $12,000 after 5 years at 5.2% monthly

a) $P = \frac{5000}{(1.03)^8} = \$3947.05$. **b)** $P = \frac{12000}{(1+\frac{0.052}{12})^{60}} = \$9257.82$.

QUESTION 4

Two accounts advertise 6%. Account A compounds yearly. Account B compounds monthly. Compare the balance after 4 years on a $3,000 deposit. Explain the difference.

Account A: $3000(1.06)^4 = \$3787.43$.

Account B: $3000(1.005)^{48} = \$3811.47$.

Account B has $24.04 more. Monthly compounding adds interest sooner, so later interest is earned on a slightly larger balance.

QUESTION 5

Find the time needed for $4,000 to grow to $6,000 at 5% compounded yearly. Give the answer to the nearest tenth of a year.

$6000 = 4000(1.05)^t$, so $t = \frac{log(1.5)}{log(1.05)} \approx 8.3 years.$

QUESTION 6

Create two investments with the same starting amount and rate but different compounding periods. Predict which grows more, then compare after 10 years.

Financial Decision Word Problems (Key)

Name Block Date

QUESTION 1

A student puts $150 at the end of every month into an account earning 4.8% compounded monthly. Use technology to find the balance after 3 years. Compare the deposits with the interest earned.

Payment amount

Number of payments

Rate each payment period

What will you compare?

For an ordinary annuity, $FV = 150\frac{\left(1+\frac{0.048}{12}\right)^{36}-1}{\frac{0.048}{12}} \approx \$5795.72$. Deposits total $5,400, so about $395.72 is interest.

QUESTION 2

Plan A charges $38 per month plus $0.08 per text after 500 texts. Plan B charges $52 per month with unlimited texts. At what monthly usage do the plans cost the same? Which plan is cheaper on each side of that point?

For $t > 500$, solve $38 + 0.08(t - 500) = 52$, giving $t = 675$. Plan A is cheaper below 675 texts; Plan B is cheaper above 675.

QUESTION 3

A laptop costs $1,600 cash. A payment plan asks for $175 at the end of each month for 10 months. The account used for the cash purchase could earn 4% per year. Compare the choices in today's dollars and name one other factor that matters.

The payment total is $1,750. Discounting monthly gives a present value of about $175\frac{1-(1+\frac{0.04}{12})^{-10}}{\frac{0.04}{12}} = \$1718.34$. Cash costs about $118.34 less. Cash flow, warranty, fees, or flexibility may also matter.

QUESTION 4

A car can be leased for $390 per month for 36 months with $2,000 due at signing and a $17,000 buyout. Or it can be bought now for $29,500. Compare total cash paid if the driver keeps the car after three years. State two important facts the totals leave out.

Lease then buy: $2000 + 36(390) + 17000 = \$33,040$. Buying now costs $29,500, which is $3,540 less before financing or investment effects. Missing factors include taxes, fees, maintenance, resale value, interest, kilometre limits, and timing of payments.

QUESTION 5

Jordan can invest \$4,000 at 5.4% compounded monthly or use it to pay down a credit-card balance charging 19.9% compounded monthly. Compare the one-year dollar change in each choice, assuming no new purchases or payments.

Investment gain: $4000\left(1+\frac{0.054}{12}\right)^{12}-4000\approx\$221.43$. Card interest avoided: $4000\left(1+\frac{0.199}{12}\right)^{12}-4000\approx\$872.77$. Paying the card saves about \$651.34 more over the year.